

EGERTON LEIGH, ESQ PE


## Dr. BROOK TAYLOR's

## Method of

## PERSPECTIVE

 Made Eafy,Both in Theory and Practice. In TWO BOOKS.

BEING
An Attempt to make the Art of Perspective eafy and familiar; To
Adapt it intirely to the Arts of Design;
AN D
To make it an entertaining Study to any Gentleman who fall chafe fo polite an Amufement.

## By Joshua KIRBY, Painter.

Illuftrated with Fifty Copper Plates; molt of which are Engraved by the Author.
The Practice [of Painting] ought always to be built on a rational Theory, of which Perspective is both the Guide and the Gate, and, without which, it is impofible to fucceed, either in Defigning, or in any of the Arts depending thereon.

Leonardo da Vinci upon Painting, p. 36.

## The SEcond Edition.

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> MDCCI.

## T 0

## Mr. H O G A R T H.

$S I R$,
TF your extenfive Knowledge and Genius in the Art of Painting did not entitle you to a Dedication of the following Theory of Perspective, the great Obligations which I am under for your Friendifhip and Favour, would claim not only this, but every other Token of my Gratitude and Affection. But this Work in a peculiar Manner has a Right to your Patronage and Protection, as it was You who firft encouraged me to write upon the Subject: And if it has any Merit, the Publick, in a great Meafure, are obliged to you for it.

I fhall not follow the common Method of Dedicators, by attempting a Panegyrick upon

## DEDICATION.

your amiable Qualifications; which might appear like Flattery, and offend your Modefty: I fall only beg Leave to fay, that your own inimitable Performances are greater Inftances of your Genius in the Arts of Defign, your Know= ledge of the Human Paffions, and your Contempt of Vice and Folly, than it is in my Power to express; and that,

$$
I a m, S I R,
$$

With the greateft Effeem and Gratitude, Your mof obliged,

And obedient Servant,

JOSHUA KIRBY.

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## P R E F A C E.

THE many Treatifes already publijbed upon Perspective, may make it appear needle/s to augment the Number ; it is therefore neceffary to give the Reafons that induced me to undertake Juch a Work.

The Defign of the following Treatife, is, by exbibiting a New System of Practical Perspective, to make this bitberto intricate, but ufeful Art, eafy and familiar to every Capacity: And to drefs it in the mof fimple Garb; that its Parts may be clearly feen, and its whole Defign (fo far as it relates to Painting, Ecc.) eafily underfood. For certain it is, that no Subject bath been treated in a worfe Manner than this, notwitbfanding the many Volumes wobich bave been zerote upon it ; fome purely Matbematical, and therefore unfit for the generality of Perfons who are concerned in the Arts of Defign; and others webolly Mecbanical , made up of incoberent Schemes, unapplicable Examples, and fuch a Confufion of unneceffary Lines, as tend only to puzzle and difcourage the Learner. My Intention, therefore, is, to feer between the abfirufe matbematical Reafoning of fome, and the tedious and falfe Explications of others; and from thence to produce a Syfem of Per/pective upon certain and fimple Principles, eafy to be underfood and applied to Practice.

This is a general Account of the following Work; which is the Product of Several Years Study and Application: And bow I bave fucceeded in the Attempt, is fubmitted to the Candour of every ingenuous Reader. But that Juch an Undertaking was neceffary, is fufficiently tefified by tbe many eminent Painters, and other curious Artifts, who perfuaded me to profecute my Defign, and have generoufly encouraged the Publication of this Work.

I bave intitled this Treatife $\mathrm{D}_{\mathrm{r}}$. Brook Taylor's Perspective, \&cc. out of Gratitude to that ingenious Autbor, for furnijbing me with Principles to build upon; and becaufe bis, thougb a very Jmall Pam-
phlet, is thought the moft correct, concije, and comprebenfive Book upon the Subject. I bave not proceeded exactly in bis Metbod; for that was not agreeable to my Plan: Nor bave I explained bis Propofitions, Theorems, Ec. in a regular Manner; fince that alfo was inconffient with the Order of my Work: But I bave had Regard to bis Principles in general, fo as to make bis Meaning more intelligible, and that kind of Perfpective of more univerfal Ufe. This Book of Dr. Taylor's was firft publijbed in the Year 1715, and was intitled Linear Perspective; and in the Year 1719 be publefbed anotber fmall Tract, which be called, New Principles of Linfar Pfrspective; and which be intended as an Explanation of bis firft Treatife. But, notzoithffanding both thefe Treatifes are jo curious and ufeful, fere bave been able to underftand bis Schemes; and when they bave underftood them, bave been as much puzzled in applying them to Practice. And in the Year 1738, Mr. Hamilton favoured the World with two Volumes in Folio, intitled Stervography, webich be has explained in the Manner of Dr. Taylor, and wbich, though a very curious Work, and worthy the Perufal of every good Matbematician, yet, I may venture to affirm, that but very few of thofe Perfons who are Students in the Arts of Defign can comprebend it; and were they qualified with a Sufficient Stock of Matbematical Knowledge, it would take up more Time than they either could, or would cbufe to Spare. However, I muft frankly acknowoledge, that I think it the beft Syfem of Mathematical Projection * that ever was, or, perbaps, ever will be, made public ; and I 乃ould be very ungenerous in not conféfing that it has been of great Service to me in feveral Parts of my Work; and that I am indebted to it for fome Things which I hould never bave thought of, bad not that ingenious Gentleman pointed them out to me; and I bope, that this publick Acknoweledgment will prevent the Imputation of Plagiarijm, and be a Jufficient Satisfaction for the Liberties wobich I bave taken with bis Work.

[^0]The Plan wobich I bave proceeded upon in the Profecution of my Defign, is this : I bave divided the whole Work into two Books; the firf I bave called A compleat System of Perspective, wobich contains the Theory and its Application to Practice; and the Second, The Practice of Perspective, which contains the practical Part only. This Metbod of treating the Subject Seemed to me the mof eligible; becaufe there are fome who do not like to take Things for granted, but chooje to be convinced by Demonflration, and to bave the Reafon of Things explained upon certain Principles. For fuch, I intend the firft Book; and for others, who either want Time or Capacity to go regularly through the thcoretical Part, I bave wrote the fecond Book; that fucb Perlons may be enabled to determine the Appearance of all Kinds of Objects upon the Picture with the greateft Eafe and Expedition: So that the whole together (if I am fo bappy as to bave fucceeded in my Attempt) may be called a compleat Sytem of Perfpective, fo far as it relates to the Art of Painting, छc.

In the firft and fecond Cbapters of the firf Book, I fuppofe my Reader a mere Novice, not only in Per/pective, but in every Thing which it is neceflary be fbould know as previous tbereto; and therefore I bave begun with an Explanation of Mathematical Inflruments, and bave flerwn their different Ujes; after which I bave explained a ferw Geometrical Definitions and Propofitions, and from thence I bave proceeded to fbew bow to defcribe (in a mechanical Manner) fuch Geometrical Figures as may occur in the following Work. And becaufe Perfpective is an optical Science, I bave given fome Sbort Abfracts from the moft eminent Writers upon Opticks; by which Means the unlearned Reader will bave a general Notion of tbe Eye and the Nature of Vifon, the Reflection and Refraction of the Rays of Light, and of the Caufe of Colours. I Jay the unlearned Reader, becaule I do not prefume to give Inftructions to Perfons who are well acquainted with the Mathematicks or Pbilojophy, but only to fuch as are ignorant in thefe Matters, but are nevertbelefs defirous of Jeeing the Foundaiions upon which Per/pective is built ; and therefore, all that bath been bitberto advanced, may be omitted by the learned Part of my Readers, as an imperfect Abliract of what they are infinitely better acquainted with than myself.

## PREFACE.

The Third Cbapter of the fame Book begins with an Introduction to Perfpective, which I bave endeavoured to explain in a familiar Manner, by fuch Objects as we are every Day converfant with; and then I bave proceeded to the Theory, which I bave ranged under the following Heads, viz, I. Of Objects which are in Planes perpendicular to the Picture; 2. Of Objects which are in Planes perpendicular to the Ground ; And 3. Of Objects which are in Planes inclined to the Ground ; becaule there are no other Situations in which Objects can be difpofed; that is, they muft be either perpendicular, parallel, or inclined: And every Example, which I bave produced, is immediately applied to Practice; and by that means the Theory and Practice are fo clofely connected, that they ferve to explain each other, and to fix both very firongly in the Memory.

And thus baving explained the Thbory and general Practice of PerSpective, I bave in the fourth Cbapter of the fame Book, confidered the different Kinds of Perfpective, viz. when the Picture is either perpendicular, parallel, or inclined. The perpendicular Pisture is what is commonly made ufe of, and is placed perpendicular to the Ground; the parallel Picture is placed parallel to the Ground, (fuch as Ceilings, and the like ;) and the inclined Picture, is fuppofed to be inclined to the Ground, and is rather more curious than ufeful. I bave next given an eafy Metbod of determining the Reprefentation of any Objects upon Domes, vaulted Roofs, and other uneven Surfaces, by means of Reticulation or Net-Work; which is all that is contained in this Cbapter. The fitth Cbapter of the fame Book contains the Perjpective of Shadorws, both in Theory and Practice; and this I bave treated in the very fame Manner as I bave done the Perfpective of Objects: So that what is contained in this Chapter, is deducible from what bath been already advanced in the third Cbapter, and follows like fo many Corollaries from thofe general Propofitions. The fixth Chapter of the fame Book contains fome general Infructions for choofing a proper Diftance for the Eye, Ec. and the bad Effects of viezoing a Picture from any other than the true Point of Sigbt; and the feventh and laft Cbapter, is principally copied from Mr. Hamilton, and contains an Explanation of Aerial Perfpective, the Chiara Ofcura, and Keeping in Pictures.-- And thus baving gone through the The-

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oretical Part, I bave proceeded to the fecond Book, wbich contains the practical Part only; in wewich I bave obferved much the fame Order and Metbod as in the Theory, and therefore that needs no farther Explanation.

The Figures I bave made cboice of, to demonftrate and explain tbis Syfem of Perjpective, are not fet off with Ornaments, to attract the Eye, but are done with Simplicity, to inform the Underftanding; and are fuch as every common Mechanick bas clear and determinate Ideas of, and confequently of the moft univerfal Ufe: For the SQuare, the Triangle, and the Circle, are not only the Foundation of moft geometrical Figures, but are alfo the fimple Materials of Sbapes in general, and of which regular Buildings, in particular, are alwayss compofed. And I may, zeithout the leaft Arrogance, affirm, that, had the feveral Writers upon Perfpective Jberwn borv to find the Reprefentations of thoje three Figures only, upon different Planes, and in various Situations, their Works would bave been moxe intelligible, and of much more Service, than they now are with fuch a Multiplicity of ornamental Schemes and unapplicable Examples. In Jhort, the Principles of Perfpective are few and Simple, and tberefore to explain them by a vaft Number of ornamental Figures, would ferve.only to divert the Eye and miflead the Fudgment, and to make that appear obfcure and difficult, which in its own Nature is extremely clear and eafy.

And bere it may not be improper to obferve, that the Learner is defired to draw out every Figure as be proceeds, which will Serve to fix them in bis Memory, and to make their Explanations more eafy and fa-- miliar to bim. It is a Method I bave always practifed myjelf, reitb Succefs; and therefore think it may be of Service to others: However, thofe who by an extraordinary Capacity: can carry on a long. Train of Ideas togetber, and can recollect, compare, and combine them as they 'pleafe, need not give themfelves this Trouble.

This is a general Account of the following Work: But before I quit the Subject, I Joall beg leave to fay Jometbing upon the Ulefulnefs of PerSpective to every Perfon that is any ways concerned in the Arts of Defign, and to recommend the Study of it in particular to every. Tyro in the Art of Painting; wbich I could wifl: might put a Stop to that Ridicule and

## PREFACE.

Contempt with which it bas been treated by a fort of People, who are too ready to condemn a Branch of Science, which they bave neglected to gain a jufficient Knowledge of. Thefe Perfons bring to my Mind a Story of Leonardo da Vinci, a famous Italian Painter wbo flouriblbed in the latter End of the fourteentb Century *. He tells us, that a Friend of bis, named Boticello, bad a peculiar Pique againft Landkips, thought tbem much beneath bis Application, and looked upon them in a mof contemptible Light: But, fays Leonardo, the Reafon was, becaufe be was a very Jorry Landjkip Painter: And our Author adds, that for tbis Reafon bis Merit in otber Matters was the lefs regarded.

That Perfpective is an effential Requifte in a good Painter, is attefted by all our moft eminent Artifts, and is moreover confirmed by almoft every Autbor + who bos wrote upon Painting; nay, the very 'Ierm Painiting implies Perjpective. For to draw a good Picture is to draw the RepreSentation of Nuture, as it appears to the Eve; and to dr wo the PerJpective Reprefentation of any Object, is to draw the Reprefent ution of that Object as it appears to the Eye: Therefore the Terms Painting and Perfpective feem to be fynonymous, though I know there is a critical Difference between the Words. Yet this will Jerve at leaft to Jhew the near Alliance between P inting and Perjpective; that if the one doth not comprebend the other, Perfpective, bowever, miy be faid to be the Bafis upon wobich Painting is built ; and iberefore be who attempts to paint a Picture witbout baving a gencral Knowoledge of it, will always wander in the Mazes of Uncertainty, be jubject to the greateft Errors, and bis $W$ orks, like tbofe of Boticello, will be the lefs regarded. And what is foid of the Ufefulnefs of Perfpective to Painters in particular, may be applid to Artifts in general; fuch as Engravers, A, cbitects, Statucries, Cb:Jers, Carvers, Ec. It will aljo be an entertaining Study to any Gentlem: $n$, who bas eitber a Tafte for Drawing, or is a Lover of Painting; as it will enable bim to draw cut the Reprefentation of any Building or Irojpect, and to form a tolerable Fudyment of a Picture witbout any otber Aljifance. I would not be underjtood to mean, that a

[^1]Perfon is always to follow the rigid Rules of Perpective, for there are fome Cafes in which it may be neceflary to deviate from them; but then be muft do it with Modefty, and for Jome good Reafon, as we bave Serwn in fome Parts of this Work. Nor would I be thought to defire the Artift to make Ufe of Scales or Compafes upon all Occafions, and to drave out every Line and Point to a Matbematical Exactnefs; no, the Defign of this Work is quite the Reverfe; it is to teach the general Rules of Perfpective, and to enforce the Practice of it by eafy and felf-evident Principles; to afijit the Judgment, and to direct the Hand, and not to perplex, either by unneceffary Lines or dry Theorems. Upon the whole: He that bas a true Genius, and will take Pains to learn the Principles delivered in this Treatife, woill be taught to See Objects with Such Exactness, and bis Fudgment will be founded upon fuch folid Principles, that be will be enabled to drawo out any Reprefentation with more Eafe, and with much more Correctnefs, than the greateft Genius who is ignorant of Perfpective, or be who defpifes the Rules of Juch a necelfary Art.


## C O N TENTS.

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Compleat System of Perspective. B OOK I.

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\begin{array}{llllllll}
\mathrm{A} & \mathrm{P} & \mathrm{P} & \mathrm{E} & \mathrm{~N} & \mathrm{D} & \mathrm{I} & \mathrm{X} .
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A Com-

## Compleat SYSTEM of PERSPECTIVE.

BOOK I. CHAP. I.

Of Inftruments used in Drawing, Geometrical Definitions and Propofitions, and Practical Geometry.

SECT. I.

Of Instruments used in Drawing.

THE Inftruments neceffary in Drawing are as follows, viz.

i. A Tee-Square. Fig. I.

2. A Parallel Ruler. Fig. 2.
3. A Drawing-Board; which is a moth Board made exactly fquare at the Corners.
4. A Sector: Fig. 3.
5. A Pair of Compaffes and a Drawing-Pen.
6. A Semi-Circle, or Protractor. Fig. 4. This Inftrumont is half a Circle, divided into 180 equal Parts, which are called Degrees.
There are all the neceffary Inftruments in Drawing, and may be had at any Mathematical-Inftrument Maker's *.
In regard to their different Ufos: They are almoft univerfal; but I hall only confider them as applied to particular Purposes; and firft, of the Tee-Square and Drawing-Board.
7. Let it be required to draw one Line $C D$, perpendicular to another Line AB.

After having fixed a Piece of Paper upon the Drawing-Board, apply the fquare Arm ED of the Tee-Square to the Side of the Board, and draw the Line AB; then lay it in the fame Manner againft the Top or Bottom of the Board, and draw a Line touching the other Line in the given Point. Thus, let D be the given

[^2]Point; then draw $C D$, and the Line $C D$ will be perpendicular to the Line AB.--And if an oblique Line AC is wanted; lay the Arm $A B$ of the Square, which turns upon a Screw $C$, againft the Edge of the Board, and move the Ruler backwards and forwards, 'till you have got it to the Inclination you want.
Fig. 2. upon a loofe Paper, without ufing a Drawing-Board: Thus, let it be required to draw one Line CD, Fig. 8, parallel to a given Line $A B$; and let $C$, or $D$, be the Diftance it is to be from $A B$ : Lay the Edge $A B$ of the Ruler, to the given Line $A B$, and keep the Limb a b, fixed; then move the other Limb cd, to the Diftance propofed, and draw a Line, as $C D$, which will be parallel to $A B$. So that having given only one Line, and erected a Perpendicular thereon, we may draw any Number of parallel Lines, or Perpendiculars to them; only obferving to fet off the exact Diftance of every Line by a Prick of the Compaffes, like C or D.
Fig. 3. 3. The SECTOR is made of two brafs Rulers, AB, AC, artificially fixed upon a Center A: This Inftrument is ufually filled with a great Number of different Scales, which, tho' very ufeful in many Parts of the Mathematicks, are neverthelefs foreign to our Purpofe; and therefore, I fhall confider it only as having what is called a Line of Lines on one Side, and a Line of Polygons on the other; which different Scales are expreffed upon the Sector, by the Letters LL and PP, as in the Figure; LL ftands for the Line of Lines, and PP for that of Polygons. The Line of Lines ferves as an univerfal Scale for dividing any Line into equal Parts, or into any given Proportion; for inftance, divide Fig 24. the Line AB into fix equal Parts: Take the Length of the Line in your Compaffes, and fet one Leg of them in the Point 6, upon the Line of Lines; then open the Sector 'till the other Leg of the Compaffes coincides with the Point 6 , which is on the Line of Lines upon the other Limb of the Inftrument; in this Pofition keep the Sector fixed, 'till you have taken the Diftance from I to I; which Diftance will be one fixth Part of the Line given. And in the fame Manner, a Line may be divided into any Number of equal Parts, even though they fhould exceed the Numbers upon the Sector; fuppofe, for inftance, it was required to divide a Line into 24 equal Parts; then fet the Length of the Line from 12 to 12 and divide the I-I2th Part into two Parts, which will anfwer the Purpofe.
Of Instruments, Ef.

The Line of Polygons is called fo from its Ufe; which is, to diwide a Circle into any Number of Parts, as in Fig. 27. which Figures are called by the general Name of Polygons; and the Method of ufing this Scale is extremely eafy. : For having firft defcribed a Circle, take the Radius (that is half its Diameter) and fet it upon thefe Lines from 6 to 6 ; in this Pofition keep the Sector fixed, and you will have a Scale for dividing any Circle of that Radius into any Number of equal Parts; for if you want a feven-fided Figure, (or Heptagon) take the Diftance from 7 to 7 ; if an eightfided Figure, (or Octagon). take the Diftance from 8 to 8 , and fo on.
4. The laft Inftrument is the Semi-Circle or Protractor, Fig. 4 which is ufed in drawing all kinds of given Angles, and in the following Manner.

Let it be required to make a right Angle* CAB, from the Fig. 7 y 1 Point A, upon the given right Line AB.--LLay the lower Side BC of the Inftrument, exactly even with the Line AB , and in fuch a manner, that the Point, or Center A, will coincide exactly with the Point A upon the Line AB; then make a prick at 90 , and draw $A C$, and the thing propofed is done.--And after the fame Manner any other given Angle may be drawn, which a little Ex: perience will make much more eafy than W.ords can do.

## S E C T. II.

Geometrical Definitions and Propositions, princio. pally from Simpfon's and Pardie's Geometry.
4. N Angle is the Inclination of two right, or ftraight Lines, AD, CD, meeting each other in a Point, as D; and the middle Letter D always denotes the Angle.
2. When one right Line $C D$, falling upon another $A B$, makes the Angles on both Sides equal, thofe Angles are called right Angles, and the Line $C D$ is faid to be perpendicular to $A B$; and if any Line AC be drawn from a Point A in one Line, to any Point C in the other, the Line fo drawn is called the Hypothenufe.

3: An acute Angle BDE, is lefs than a right Angle BDC.
4. An obtufe Angle ADE is that which is greater than a right Angle ADC.
5. Parallel Lines are fuch as are equally diftant from each Fig. other, as AB, CD.

[^3]6. A
6. A plane Figure is that which lies evenly between its Bounds or Extremes; thus any fmooth Surface is a plane Surface, and is therefore called a Plane.
Fig.9,10. 7. All plane Figures bounded by three right Lines, $A B, A C, B C$, are called Triangles.
Fig. 10. 8. An equilateral Triangle ABC, is that whofe Bounds or Sides: are all equal.
Fig. 11. 9. Every plane Figure $A B C D$, bounded by four right Lines, is called a Quadrilateral; and if its Sides and Angles are equal, it is called a Square.
10. Any quadrilateral Figure, whofe oppofite Sides are parallel, but not equal, is called a Parallelogram.
Fig. 12. II. A right Line is faid to be perpendicular to a Plane when it: ftands on it at right Angles; thus the right Line EF, is perpendicular to the Plane $A B C D$, when it ftands like a Pillar upon the Ground, and is inclined no more to any one Side of the Plane than to the other.
Fig. 13. 12. One Plane $A B C D$, is right and perpendicular to another EF, when, like a well-made Wall, it inclines and leans on one Side no more than it does on the other.
Fig.9,10. 13. Two right Lines, if they meet fo as to cut or crofs each other, are in the fame Plane; wherefore all the Angles, A, B, C, and Sides $\mathrm{AB}, \mathrm{BC}, \mathrm{CA}$, of every Triangle, are in the fame Plane.
Fig. 14. 14. If two Planes ABC, EFGH, cut or interfect one another, they fhall do it in a right Line EF, which Line is called their Common Section.
15. If a right Line FG, be perpendicular to two right Lines FD, FE, which are in the fame Plane $A B C$, that Line is alfo perpendicular to that Plane.
16. If a right Line $F G$ be perpendicular to three right Lines, FI, FE, and FD; thofe three Lines are all in the fame Plane, ABC.
17. If two Lines FG, EH, are perpendicular to the fame Plane $A B C$, they will be parallel to one another.
Fig. 15. 18. Two Lines EG, FH, perpendicular to the fame Plane ABCD , cannot be drawn through the fame Point $G$.
Fig. 16. 19. If two parallel Planes. ABCD, EFGH, are cut by a third Plane IKLM, the common Sections, OP, QR, are parallel.
Fig. 17. 20. If the Lines GM and HN, are divided by parallel Planes, then GI will have the fame Proportion to IM, as HL has to LN; and the Section MN, IK, of any plane Triangle MGN, by two parallel Planes, is always in a given Ratio; that is, IK is in the fame Proportion to $I G$, as $M N$ is to $M G$.
21. A folid Angle E, is made by the meeting of three or more Fig. 38: Planes, and there joining in a Point ; like the Point of a Diamond, or the Corner of a Die, or Cube.
22. If we imagine a Line, as EB, fixt above in the Point $E$, to be moved along the Sides of any regular Figure, $A B C D$, that Line, by its Motion, will defcribe a Figure that is called a Pyramid.
23. If a Line faftned as before, move round a Circle, AB , it Fig. Ig: will defcribe a Figure that is call'd a Cone; and the Circle is its Bafe, and a Line drawn from the Vertex C, to D, is called its Axis.
24. If a Line AD, move uniformly about two angular Figures, Fig. 20: $A B C, D E F$, which are every Way equal, having their Sides and Angles mutually parallel and correfponding exactly to one another, as $D F$ to $A B, D E$ to $A C, \Xi_{c}$. then that Line by its Motion fhall defcribe, if it hath three Sides, a Prifm; if four, a Cube or Parallelopiped.
25. If a Line move uniformly round two equal and parallel Fig. 27: Circles, it fhall defribe or generate a Cylinder; and the Line joining the Centers $A B$, in the two Bafes, is called its Axis.
26. If any folid Body is laid with one Face upon another Plane, Fig. 22. the Space which that Face takes up is called its Seat; thus the Cube CG, refts with its Face CDEH, upon the Plane IK ; therefore, CDEH is the Seat of the Cube on that Plane; and thus the Points C and D , are the Seats of the Lines $\mathrm{CA}, \mathrm{DB}$; as are alfo $\mathrm{CD}, \mathrm{DE}$, the Seats of the Lines AB and BF ; and fo likewife, the Seat of the oblique Line DF , is the Line DE .
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\text { SECCT. III. } \\
\text { Of PRACTICAL GEOMETRY. }
\end{gathered}
$$
\]

To erect a Perpendicular CD , from D , near the Middle of a right: Fig. s . Line AB.
FROM D, fet off on the Line $A B$, any Diftance $D A, D B$, Arc cd, and ach other; then from $A$, defcribe at Pleafure the and then from the Po fame Diftance from B, defcribe the Arc ab; fo will $C D$ be perpendicular to $A B$.
To let fall a Perpendicular CD upon a Line AB , from a Point C , Fig. 6 . without the Line.
From the given Point C, defrribe the Arc AEB at pleafure, and from the Points A and B, defcribe two other Arcs cutting each other in F ; draw CF , then are CD and FD perpendicular to AB .

Fig. 7. To ereet a Perpendicular AC, upon the Extremity A, of a Line AB. With any Diftance, defcribe the Arc efg from the given Point A, and fet off the fame. Diftance upon the Arc from e to $f$, and from $f$ to $g$; then from the Points $g$, $f$, defcribe the Arcs ab, $c d$; and from their Section C , draw CA , which will be perpendicular
Eig. 8. To drave one Line CD, from a given Point D, parallel to a given
Draw, as you think proper, the oblique Line $A D$ from the given Point $D$, cutting $A B$ in $A$; and from the Point $A$, with the Diftance $A D$, defcribe the Arc $D B$, cutting $A B$ in $B$; then from $D$, with the Diftance DA, defcribe the Arc AC; and make AC to $A D$ to $B D$; and then draw the Line $C D$, which is parallel to AB.

Or it may be done yet eafier, by defcribing two Arcs C, D, with the fame Radius as in the other Figure, and drawing the Line,
Fig. 9. To make any Triangle, as ABC , from tbree given Lines, $\mathrm{AF}, \mathrm{BE}, \mathrm{CD}$.
Draw a Line $A B$ at Pleafure, and make $A B$ equal to the given Line AF; then from the Point $B$, with the Radius $B E$, defribe an Arc cd; with the Radius CD, from the Point A, defcribe another Arc a $b$, cutting the former Arc in $C$; then from $C$ draw Lines to A and B ; and then will ABC be a Triangle whofe Sides, are refpectively equal to the given Lines $A F, B E$, and $C \cdot D$.
Fig.to:
To make an equilateral Triangle upon a Line given, AB .
Take the Length of $A B$ in your Compaffes, and from the Points $A$ and $B$, defcribe two Arcs cutting each other in $C$; then from $C$, draw $C A, C B$; and then is $A B C$ an equilateral
Triangle.

Fig.ve: To make Geometrical Square $A B C D$, on the given Line $A B$. From the Point $B$ erect the Perpendicular BC; from $B$, with the Radius $A B$, defcribe the Arc AC, cutting the faid Perpendicular in $\mathrm{C}_{\text {; }}$ from A and C , (with the fame Radius) defcribe two more Arcs cutting each other in D; then draw DA, DC, and the Figure propofed is compleated.
4ig. 23. Io make an Angle, with the Line AB, equal to a given Angle X. From the Point A, with any Radius, defcribe the Arc cd; froms D, with the fame Radius, defcribe the Arc $a b$; take the Length,

## I.





| D | 8. |
| :---: | :---: |
| B | C |
|  |  |
|  |  |
|  |  |



of $a b$, and transfer it from $c$ to $d$, and through d draw a Line to $A$; then is the Angle $B A C$ equal to the given Angle $a D b$.

To bifect, or divide an Angle A into two equal Parts. Fig. 23:
From the Point A, with any Radius, defribe the Arc BC; di- No. $\boldsymbol{m}_{\text {: }}$ vide $B C$ into two equal Parts, and draw $A D$.

To divide a right Line AB into any Number of equal Parts. Fig. 24:
From the Point A draw at pleafure the Line AC, and make BD parallel thereto; then carry as many equal Parts along the Line AC , from the Point A , and along the Line BD, from B , as you would divide the Line AB into (for inftance, fix Parts) and draw the tranfverfe Lines, which will divide the propofed Line as was required.

Or, it may be done by drawing a Line $A B$, parallel to a given Fig. 25: Line CD; then by fetting as many equal Parts upon the Line AB as CD fhould be divided into, and by drawing Lines from thence to a Point, as E, from every Divifion, and in fuch a Manner, that the outward Lines AE, BE, fhall touch the Ends of the Line CD, as in the Figure. I fay then, the Line CD will be divided into fix equal Parts.

To infcribe a Circle within a Square ABCD. Fig. 26 .
Draw the Diagonals AD, BC, and where they crofs each other will be the Center of the Square, which confequently is the Center of the Circle alfo.

To inforibe a Square in a given Circle.
Fig. 26.
Draw the Diameter $A B$, from $A$ and $B$ defcribe the Arcs $a, b$, No. 2 . and draw DE ; from $\mathrm{A}, \mathrm{D}, \mathrm{B}, \mathrm{E}$, draw Lines as in the Figure, which will be the Square required.

As to the Geometrical Conftruction of Polygons, I fhall not Fig. 27. take up the Reader's Time about them; for they may be defcribed very eafily by Means of the Scale upon the Sector for that Purpofe, as has been obferved before, under the Word Sector.

C HAP.

## CH A P. II.

Of the Eye and the Nature of Virion, the Reflection and Refraction of the Rays of Light, and the Cause of Colours.

SE CT. I.

Of the Eyb and the Nature of Vision.

THE Defign of this Chapter is, to explain to the unlearned Reader the Construction of the Human Eye, and to give him a general Idea of the Nature and Cafe of Vifion; and not to proceed in a regular Manner upon Opticks, but only to take Notice of fome particular Parts of it, by which he will be enabled to fee more clearly the Nature of Perfective. In order to which, I fall take Quotations from the molt eminent Writers upon that Subject, and not prefume to give him much of my own; as nothing which I can offer will be new, or fo much to the Purpofe.
ci Every vifible Boảy emits or reflects inconceivably fall Par" tiles of Matter from each Point of its Surface, which iffue from " it continually (not unlike Sparks from a Coal) in frat Lines si and in all Directions. There Particles entering the Eye, and "cs 'ftriking upon the Retina (a Nerve expanded on the back Part of "s :the Eye to receive their Impulfes) excite in our Minds the Idea " of Light, and as they differ in Magnitude, they produce in us " the Ideas of different Colours.
"T That the Particles which conftitute Light, are exceedingly "f fall, appears from hence, viz. that if a Hole be made through "s a Piece of Paper with a Needle, all the Rays of Light which " proceed at the fame Time from all the Objects on one Side of it, " are capable of paffing through it at once without the leaf Con" fufion; for any one of thole Objects may as clearly be feen " through it, as if no Rays paffed through it from any of the "reft. Further, if a Candle is lighted, and there be no Obftacle " in the Way to obftruct the Progress of its Rays, it will fill all the Space within two Miles of it every Way with luminous sr Particles, before it has loft the leaft fenfible Part of its Substance st thereby.
"Diftance into nine Squares, feverally equal to the Square bd, as cc. reprefented in the Figure. The Light then which falls upon "r the Plane bd, being fuffered to pafs to a double Diftance, will. as be uniformly fpread over four times the Space, and confequently "s will be four times thinner in every Part of that Space, and at ss a treble Diftance will be nine times thinner, and at a quadruple
" Diftance fixteen times thinner than it was at firft ; and fo on or according to the Increafe of the fquare Surfaces bcde, $B C D E$, " $\mathcal{E}_{c}$. or of the fquare Surfaces Abfg, ABFG, छc. built upon of the Diftance $\mathrm{Ab}, \mathrm{AB}, \mathrm{\Xi}^{\circ}$. Confequently the Quantities of this © $s$ rarified Light, received upon a Surface of any given Size or "Shape whatever, removed fucceffively to thofe feveral Diftances, "s will be but one quarter, one nirath, one fixteenth, of the whole © Quantity received by it at the firft Diftance Ab . Or in general "Words, the Denfities or Quantities of Light, received upon any " given Plane, are diminifhed in the fame Proportion as the «Squares of the Diftances of that Plane from the luminous Body 6 are increafed; and on the contrary, are increafed in the fame "Proportion as thofe Squares are diminifhed. For the Lights of or the feveral Points of the Body, which feverally follow this Rule, "s will compofe a Light which will ftill follow the fame Rule."*

Having thus far explained what we are to underftand by the Rays of Light, we will now proceed to a Defcription of the Human Eye, and confider the Nature of Vifion.
4ig. 28. " ATYC is the Reprefentation of an Human Eye, diffected " through its Axis $\psi$, all the Parts being twice as big as the Life. "Here the tranfparent Coat, called the Cornea, is ABC; the «Remainder AT YCC being opake, and a Portion of a larger "Sphere. Within this outward Coat Anatomifts diftinguifh two «s others; the innermoft of which is called the Retina, being like «s a fine Net, compofed of the Fibres of the Optick Nerve Y V'T " woven together, and is white about the Parts p, q, r, at the " Bottom of the Eye. The Cavity of the Eye is not filled with " one Liquor, but with three different Sorts. That contained in "the outward Space ABCOEGFD is called the Aqueous Hu"s mour, being perfectly fluid, like Water; the other, contained

[^4]48 in the inward Space EpqrDFG is a little thicker than the " White of an Egg, and is called the Vitreous Humour; the third: "Humour, FG, is fhaped like a Lens* of unequal Convexities, " lying between the two former, and fixed to the fide Coats by "Filaments or Threads extended all round it, and is called the "Cryftalline Humour, being hard like the White of an Egg " boiled, but as clear as the other two, and differs from them in " a greater Degree of Refractive+ Power; whereby: the Rays that "came from the Points $\mathrm{P}, \mathrm{Q}, \mathrm{R}$, having received a Degree of "Convergence $\ddagger$ by the Refraction of the Cornea ABC, are " made to converge a little more by other. Refractions at the Sur"faces of the Cryttalline FG; fo that uniting in as many Points " $\mathrm{p}, \mathrm{q}, \mathrm{r}$, upon the Retina, they reprefent the Points of the Ob" ject $P, Q, R$, from whence they came." ||

The Picture of an Object upon the Retina being produced much in the fame Manner as a Picture by a Lens, viz. in both Cafes by Means of the Refraction of the Rays of Light, we will therefore, firf fhew how by the Paffage of thofe Rays through a Lens, a Picture may be produced; as this will be one confiderable Step towards explaining the Nature of Vifion: For which Purpofe I fhali quote an Experiment from the incomparable Sir Iface Nerwton. § " Let PR reprefent an Object without-doors, and AB a Lens Fig. 2g: "placed at a Hole in the Window-fhutter of a dark Chamber, " whereby the Rays that come from any Point $Q$ of that Object, " are made to converge and meet again in the Point q ; and if a "Sheet of white Paper be held at q, for the Light there to fall " upon it, the Picture of that Object PR will appear upon the "Paper in its proper Shape and Colours. For, as the Light " which comes from the Point $\mathrm{Q}_{2}$, goes to the Point q , fo the "Light which comes from other Points P and R of the Object, " will go to fo many other correfpondent Points $p$ and $r$; fo " that every Point of the Object fhall illuminate a correfpondent "Point of the Picture, and thereby make a Picture like the Object

[^5]" in Shape and Colour, this only excepted, that the Picture that " be inverted. And this is the Reafon of that vulgar Experiment " of cafting the Species of Objects from abroad upon a Wall, or "Shect of white Paper in a dark Room.*
"In like Manner, when a Man views any Object PQR, the "Light which comes from the feveral Points of the Object is fo "refracted by the traniparent Skins and Humours of the Eye, " (that is, by the Cornea ABC , and by the Cryftalline Humour
(c FG) as to converge and meet again in fo many Points in the
"c Bottom of the Eye, and there to paint the Picture of the Ob-
"喿ct upon the Retina. And thefe Pictures, propagated by Mo-
"tion along the Fibres of the Optick Nerves into the Brain, are
"s the Caufe of Vifion. For accordingly as thefe Pictures are per-
"fect or imperfect, the Object is feen perfectly or imperfectly.
16 If the Eye be tinged with any Colour (as in the Difeafe of
"the Jaundice) fo as to tinge the Pictures in the Bottom of the
"Eye with that Colour, then all Objects appear tinged with the
"fame Colour. If the Humours of the Eye by old Age decay,
"fo as by fhrinking to make the Cornea and Coat of the Cry-
"ftalline Humour grow flatter than before, the Light will not be
"refracted enough, and for want of a fufficient Refraction will
"s not converge to the Bottom of the Eye, but to fome Place be-
"yond it, and by confequence paint in the Bottom of the Eye a
"confufed Picture.---This is the Reafon of the Decay of Sight in
" old Men, and shews why their Sight is mended by Spectacles.
\&For thefe Convex Glaffes (or Lenfes) fupply the Defect of
$\because$ Plumpnefs in the Eye, and by increafing the Refraction make
st the Rays converge fooner, fo as to convene diftinctly at the
"Bottom of the Eye, if the Glafs has a due Degree of Convexity.
"And the Contrary happens in fhort-fighted Men, whofe Eyes
" are too plump. For the Refraction being now too great, the
"R Rays converge and convene in the Eyes before they come at the
"Bottom; and therefore the Picture made in the Bottom, and the
*V Vifion caufed thereby, will not be diftinct, unlefs the Object be
"s brought fo near the Eye as that the Place where the converging
"Rays convene may be removed to the Bottom, or that the
"r Plumpnefs of the Eye be taken off, and the Refractions dimi-

[^6]66 niflid

* nifhed by a Concave-Glafs of a due Degree of Concavity; or * laftly, that by Age the Eye grows flatter, 'till it comes to a due. "Figure: For fhort-fighted Men fee remote Objects beft in Old "Age; and therefore they are accounted to have the moft laft". ing Eyes." *
"As to the Diftinctnefs of Vifion in a perfect Eye, that evi" dently depends upon the Refraction of the Rays; and it is then " as diftinct as poffible, when the Refraction is fo made, as that " all the Rays which come from one and the fame Point of the "Object, meet together exactly in one and the fame Point of the
"Bottom of the Eye: But this is never precifely fo, but in thofe
"Rays which come from that Point of the Object which is at the
"Extremity of the optick Axis Qq ; for it is evident, that thofe
"Rays which come from the other Points, are reunited fo much
" the lefs exacily one than the other, as they are more diftant
"from this Axis; wherefore we cannot have at the fame time,
" the moft diftinct Senfation but in this Place alone, and the reft "will be more confufed." +

The farther diftant the Eye is from an Object, fo much lefs. Fig. $3^{\circ}$. will the Picture of that Object be upon the Retina: For let E be an Eye viewing the feveral Objects AB, CD, EF, at the Diftance OQ, OR, OS.---Having drawn the feveral Rays $\mathrm{Aa}, \mathrm{Bb}, \mathrm{Cc}$, $\mathrm{Dg}, \mathrm{Ee}, \mathrm{Ff}$, through the Pupil O , it will be manifeft, that the Picture of the neareft Object AB, will be painted at the Bottom of the Eye in the Space ab, the Object CD in the Space cg, and the fartheft Object, EF, in the Space ef; therefore, as the Space $a b$, is larger than the other two, cg or ef, the Picture of $A B$ will be larger than the Picture of the other two Objects CD or EF, which are at a greater Diftance from the Eye; and thefe Pictures will be to each other, as the feveral Diftances OQ, OR, OS, are to each other: $\ddagger$ From hence then we may eafily conceive, that the Eye may be fo far removed from the Object, that at laft the Image of that Object will totally difappear. ||
"But the Degree of Brightnefs of the Picture of an Object " painted upon the Retina continues the fame, at all Diftances be* tween the Eye and the Object, provided none of the Rays be

[^7]" Ropt
"ftopt by the Way, and that the Pupil does not alter its Aperture:-
"For inftance, when the Eye approaches as near again to the:
"Object, the Picture upon the Retina becomes double in Length
" and double in Breadth, and confequently quadruple in Surfaces.
" for the Surface would be double, if its Length alone or Breadth
" alone was double. The Quantity of Rays received through the
" fame Aperture of the Pupil, at half the Diftance from the Object,
" is alfo quadruple, and being equally fpread over four times the
"Quantity of Surface of the Retina, they are juft as denfe as
" before, when the Object was at twice that Diftance.
"It follows then that the faint Appearance of remote Objects,
" is owing to the Opacity of the Atmofphere, which hinders Part of
"their Light from coming to the Eye. Accordingly we find that
" the Sun, Moon, and Stars, appear very faint when near the
"Horizon, and brighter continually as they rife higher; becaufe
" the Tract of Vapours which lies in the Way of the Rays, is
" longeft and thickeft near the Horizon; and becomes thinner and
" fhorter as the Objects rife higher, and confequently does lefs
" obftruct the Paffage of the Rays.*
"Parallel Lines feen obliquely, as ABC, DEF, appear to con-
"verge more and more as they are farther extended from the Eye.
"Becaufe the apparent Magnitudes $\dagger$ of their perpendicular Inter-
" vals $\mathrm{AD}, \mathrm{BE}, \mathrm{CF}, \mathcal{E} c$. are perpetually diminifhed. And for
" the fame Reafon they appear to converge towards an imaginary
"Line, OG, drawn from the Eye parallel to them.
"This is the Reafon that the remoter Parts of a Walk or Floor
" appear to afcend gradually, and the Ceiling to defcend towards "the Horizontal Line OG: And that the Surface of the Sea, feen " from an Eminence, appears to afcend gradually in going from " the Shore; and that the upper Parts of very high Buildings feem " to lean forward over the Eye below, becaufe they feem to ap" proach towards a vertical Line OG.
Fig. 32. "The apparent Magnitude of a given Line, AB, feen very ob" liquely at a given Diftance, OA, increafes and decreafes in pro" portion to the Increafe and Decreafe of OP, the perpendicular
" Diftance of the Eye, from the Line AB produced; provided "the Line AO be very large in comparifon to AB. For let the "Ray BO cut a Line AC perpendicular to AB in C ; and while

[^8]III.


\& the Eye is raifed or depreffed in the Perpendicular OP, the Line "AC will increafe and decreafe as OP does, and fo will the "Angle AOC fubtended by AC, and this Angle meafures the " apparent Magnitude of AB.
"Hence the apparent Magnitudes of equal Parts $A B, a b$, of "a Line PAb, feen very obliquely at great Diftances from the "Eye, are reciprocally in a duplicate Proportion of thofe Dif"tances. For Example, let Ob be double of OB , and the " Angle OBP will be double of ObP, and accordingly fince $A B$, " a b, are equal, the Perpendicular AC will be double of ac, and " being feen twice as near as ac, will appear four times bigger "t than ac. Again, if Ob be treble of OB , the Line AC will be " treble of ac, and being feen three times nearer than ac, will " appear nine times bigger than ac, and fo oń.
"Hence the apparent Magnitude between a Row of Columns " are diminifhed in a greater Proportion than their Heights.
"The quick Diminution of the apparent Magnitudes of the " remoter Parts of long Lines or Diftances, is the Caufe of great " Difficulty and Uncertainty in our Eftimate of their Quantities. "For be the Differences of reveral Diftances or Heights never fo "s great in themfelves, they will become invifible at laft by reafon " of the Smallnefs of the Angles they fubtend at the Eye, occa"fioned by their Obliquity; and then thofe unequal Heights and "Diftances will appear equal." *

## S E C T. II.

Of the Refiection and Refraction of the Rays of Light.
er THEN a Ray of Light falls obliquely upon a fmooth po- Fig. 34. " $ل$ lifhed Surface, it is turn'd out of its Way either by Re"flection or Refraction in the following Manner. Imagine the
"Paper upon which this Figure is drawn to be perpendicular to " the Surface of ftagnating Water, and to cut it in the Line RS, "s and that a Ray of Light, coming in the Air along the Line "AC, falls upon RS at the Point C. Then fuppofing the Line "PCQ to be perpendicular to the Surface of the Water, if the * Ray be reflected, or turn'd back at C into the Air again, it will
defcribe a ftraight Line $C B$, inclin'd to the perpendicular $C P$ at an Angle PCB exactly equal to the Angle PCA, and therefore the Angle of Reflection is always equal to the Angle of "Incidence.
Pig. 35: "But if the Ray that came along A C goes into the Water at
" C , it will not proceed fraight forward, but being refracted or " bent at C, it will defcribe another ftrait Line CE, inclined to
"the Perpendicular CQ, at a leffer Angle ECQ, than the Angle ACP; and the Line CE, will always be fo fituated, that when any Circle, defrribed about the Center C, cuts the Line CA in A, and CE in E, the Perpendiculars AD, and EF, drawn from $A$ and $E$ to the Line PQ, fhall always bear the fame Proportion to each other, whatever be the Magnitude of the Angle ACP. In Water the Line EF is always threequarters of AD.
"In both thefe Cafes the Line AC is called the incident Ray,
CB the reflected Ray, CE the refracted Ray, C the Point of Incidence, PCQ the Perpendicular (at the Point) of Incidence, the Angle A CP the Angle of Incidence, BCP the Angle of Reflection, ECQ the Angle of Refraction; the Line AD the Sine of Incidence, that is, of the Angle of Incidence; and EF the Sine of Refraction, that is, of the Angle of Refraction. "As Rays of Light are inceffantly thrown out and difperfed in all poffible Directions from every Point of a luminous Body; fo when they illuminate other Bodies, on which they fall, they are alfo inceffantly thrown back from every Point of thofe Bodies. For the Points of opake Bodies fo enlightened, are vifible to the Eye at any Point of Space and in any Point of Time, as well as the Points of the luminous Body that enlightened them. The numberlefs Rays which flow from all vifible Bodies, called Objects, may be methodically diftributed in this Manner. The Surface of the Object is confidered as confifting of Phyfical Lines, and thefe Lines as confifting of Phyfical Points, and thefe Points are conceived to radiate all manner of Ways. It is ufual to make ufe of nothing elfe for " an Object but a Phyfical Line. For by how much that Line is " increafed or diminifhed in apparent Magnitude, or Brightnefs, " or Diftinetnefs, fo mach the Diameter, or Length, of any Ob" ject, in its Place, would be increafed or diminifhed.
Fig. 36. "The Point Q, from which Rays diverge, or towards which " they converge (being made to go back towards the fame Point, "thougb
$5 \pi$ as in both Cafes any Parcel of thefe Rays as QB or QB A os confidered apart from the reft, is called a Pencil of Rays. " This Figure reprefents the Manner in which the Rays of a Pen${ }^{\circ}$ cil, QA. B, diverging from any Point of an Object $Q$, and falling upon a frait Line ABC , or upon a polifhed Plane reprefented by it, do all diverge after Reflection as if they came " from another Point $q$. The Ray Q C, which falls perpendicu" larly upon the Plane $A B$, is reflected back again along the " fame Line CQ ; but all the reft falling upon it with greater "s and greater Degrees of Obliquity, as the Points of Incidence lye " farther and farther from C, are alfo reflected with Degrees of " Obliquity refpectively greater. It will feem reafonable therefore, " efpecially by attending to the Figure, that the reflected Rays, " produced backwards, (hould meet the Perpendicular QC, pro" duced in a Point $q$, fituated as far from the reflecting Plane on " one Side, as $Q$ is on the other: And confequently that all the "Rays flowing from a fingle Point $Q$, will after Reflection di" verge from a fingle Point q, at an equal Diftance on the other
"Side of the reflecting Plane.
" On the contrary, if q be a Focus to which the incident Rays
" are made to converge, the Point $Q$ will be their Focus after
" Reflection from the Surface ACB.
"What has been faid of the Point $Q$, is applicable to every
" other Point of an Object PQR; namely, that as the Focufes
" $\mathrm{Q}, \mathrm{q}$, lie at equal Diftances on each Side of the reflecting Plane,
"fo the Focufes P, p lye on each Side at other equal Diitances,
" and $R, r$ at other equal Diftances, in Lines $P$ p, $R$ r, drawn
"perpendicularly through the Plane $A B$. Hence it is eafy to
"underftand by Infpection of the Figures, that thefe Focufes
" $\mathrm{p}, \mathrm{q}, \mathrm{r}$, with innumerable others, lying all in the fame Order as

* the correfponding Points $P, Q, R$, compofe an imaginary Line of
" the fame Length and Shape as the Line PQR; and that the
"Situation of the Line pqr, with refpect to the back fide of the
" reflecting Plane, is the very fame as that of PQR with refpect
" to the fore fide of it. This Line pqr is called an Image or
"Picture of the Object PQR."*
This may fuffice to thew the Nature of the reflected Images of Objects from polifh'd Planes; the Knowledge of which is abfo-

[^9]Iutely necelfary in feveral Parts of Painting, efpecially in Landfips; where Water is often introduced; the Tranfparency of which, depends upon giving the Reprefentation of that Fluid its true or local Colour, and in giving the Reflections their proper Depths and Appearances.-.-Proceed we now to a farther Confideration of the Refraction of the Rays of Light, as introductory to the Caufe of Colours.

In the $35^{\text {th }}$ Figure we obferved, that if a Ray of Light went out of Air into Water, it would not proceed ftrait forward, but be bent and turned out of its direct Courfe at the Point of Incidence C; and that the Reafon of this Refraction, or bending of the Ray, was owing to its paffing out of a rarer or thinner, into a denfer or thicker Medium; and in Proportion as this Medium into which the Light enters, is more or lefs denfe, the Ray will be more or lefs refracted. Now what is faid of one Ray, will hold equally true as to any Number of Rays: But fince the Rays of Light are not alike, but diffimilar, fome greater and others lefs, they will be differently refracted at their Exit out of one Medium into another Medium; and being thus feparated, each Species of Rays will exhibit a Colour peculiar to itfelf; which is the Subject of the next Section.

## S E C T. III. Of the Cause of Colours.

"THE Sun's Rays are not homogeneous (that is alike) but of different Kinds, and each Sort has a different Degree of " Refrangibility; that is, in paffing through a denfe Medium they
"c are differently difpofed to be refracted, being bent or turn'd
"s out of their firft Courfe to different Diftances from the Perpen-
"dicular; and thefe feveral Sorts of Rays have each a peculiar
*Colour, viz. thofe which are leaft refrangible, are Red; the
" fecond Sort, Orange; the third Sort, Yellow; the fourth Sort,
"Green; the fifth Sort, Blue; the fixth Sort, Indigo; and the fe-
"s venth Sort, Violet, which laft are moft refrangible, or refracted
" to the greateft Diftance from the Perpendicular.
Fig. 40. To illuftrate this Matter, let GF reprefent a Parcel of the "Solar Rays entering through the Hole H of a Window-Shutter, " into a darkened Room, and there let them fall on the Prifm
"ABC, in the Point F: In paffing through the Prifm they will be
*s Severally refracted in a different Degree, and thus feparated from
$\because$ each

es each other, fo that at their Exit on the other Side at E, they *s will proceed at different Diftances from the Perpendicular EP os to the other Side of the Room; where they, will make a long " and various-coloured Image of the Sun XY; which is, perhaps, " one of the moft furprizing and agreeable Spectacles in Nature. "The feveral Sorts of Rays, after they are refracted, appear in "their: own proper Colours, in Order as follows, viz. Thofe "s which are leaft refracted, or fall neareft the Perpendicular EP, " are Red, and make the red Part of the Spectrum at R ; the " next are the Orange at $O$, the Yellow at $\bar{Y}$, the Green at $G$, "the Blue at B, the Indigo at I, and the Violet at V : And there "feven are all the original fimple Colours in Nature; and of " which ${ }_{2}$, by various Mixtures, all others are compounded, in the "common Refractions and Reflections from natural Bodies. * "From hence then we may conceive, that Colour is a Senfation "s produced in the Mind, by the Impreffion made in the Eye, by "certain Kinds or Sorts of Rays of Light, feparated from others " by means of their different Refrangibility and Reflexibility, " whereby they are divided into feveral Parcels, each endowed with " its own diftinct colour-making Power. And Bodies, whofe Sur" faces are difpofed to reflect one kind of thefe Rays more copioully' "t than any others, exhibit, and are faid to be of that Colour " which is peculiar to the Rays they moft copioufly reflect; and " the infinite Diverfity of Bodies, and the different Mixtures and " Modifications of different colour-making Rays thereby occa" fioned, muft therefore produce that infinite Variety of Colours " which beautifies the Face of Nature." $\dagger$

[^10]
## CHAP. III.

## The Theory of Perspective。

## S E C T. I.

An Explanatory Part, by Way of Introduction.

PErspective is the Art of drawing upon any Surface the Reprefentation of Objects as they appear to the Eye: In order to which, it is neceffary to fuppofe the Light fhould come from every Part of the Reprefentation in the very fame Manner, and with the very fame Strength of Colour, as it would do from the real Objects themfelves, were they put in the Place of the Picture ; becaufe then, the Eye will not be able to judge, whether what it fees be a few Colours artificially laid upon a Canvas, or the real Objects themfelves in the fame Situation.

This is a general Definition of that kind of Perfpective I am going to explain; which, is only what relates to the Arts of Painting and Defigning; but not to any of the Mathematical Arts, which are too abftrufe for my Speculations, and would be of no real Service to thofe for whofe Ufe this Work is chiefly intended: And although Perfective Reprefentations may be drawn upon any Surfaces, be they ever fo irregular, yet I fhall firft confine myfelf to fmooth even Planes, fuch as a Canvas, Wall, Cieling, or the like.---This being premifed,
Fig. 4r. Let E be the Eye, HE its Height from the Ground OP, and TOSX a fquare Object laid flat upon the Ground. Now it is evident, from what was faid in the laft Chapter, Sect. I. that the Eye will fee the 'Object TOSX, by means of the Rays of Light which come from every Part of the Object to the Eye. Let us therefore fuppofe a tranfparent Plane GLPP, like a Glafs-Window, to be fixed perpendicularly upon the Ground OP, between the Spectator HE, and the Object TOSX; and it will be as evident, that the Rays TE, OE, SE, and XE, will be cut by the tranfparent Plane GLPP, in the Points tosx; which Points are called the Projection, or in other Words, the Perfpective Reprefentation of the correfponding Points TOSX, of the original Object.* And if Lines are drawn from the feveral Points tos x , fo as to join each ather, the Figure fo defcribed, will be the Projection, or Perfpec-

[^11]tive Reprefentation, of the whole original Figure TOSX, upon the Picture.

In like Manner, fuppofe TOSX to be raifed perpendicular to Fig. 42. the Ground OP, and parallel to the Picture, but every thing elfe remaining in the fame Situation as in the former Figure; then will tosx be the Reprefentation of TOSX: For it is the Section of the Picture with the Rays TE, OE, SE, and XE, which come from the original Object to the Eye. And here let us obferve, that when the original Object is parallel to the Picture, its Reprefentation, tosx, will not only be parallel to the Original, but exactly like it, though fmaller in Proportion as the original Object is farther from the Picture; and if the Original be brought to $G$, fo as to coincide, or touch the Picture, then the Reprefentation will be equal to the Original: But on the contrary, the Original may be fuppofed fo far removed from the Picture, that the Angles, which the Rays fubtend at the Eye, growing fmaller and fmaller continually, it will at laft totally difappear, and confequently its Reprefentation upon the Picture will difappear alfo. Again, when the Original is brought to coincide with the Picture, the Reprefentation of TX will not only be equal to the Bottom of the Original, but will be at the Bottom of the Picture, in the Line GL, which is its Section with the Ground Plane OP: But as the Original is removed farther and farther from the Picture, the Reprefentation will rife higher and higher, 'till at laft, the Original being fuppofed at an infinite Diftance, its Reprefentation will vavifh into an imaginary Point C , exactly as high above the Bottom of the Picture as the Eye is above the Ground, or original Plane OP , upon which the Spectator, the Picture, and the original Object are now fuppofed to ftand. And fo alfo in regard to Objects that lie flat upon the Ground; when their Sides are parallel, then the Reprefentations of thofe Sides will be parallel alfo: Thus the Reprefentation $t x$ of TX, and os of OS, are parallel to their Fig. 41. Originals, but feverally diminifhed in proportion to their Diftance f:om the Picture; and therefore the Reprefentation of their oblique Sides TO, XS, which muft join tx, os, to compleat the Reprefentation of the whole original Figure, cannot be parallel to their Originals, but will be oblique in the Picture, and would, if continued towards the Top of the Picture, converge into an imaginary Point C, exactly as high above the Bottom of the Picture, as the Eye is above the original Plane OP. Now thefe Points, into which we fuppofe the Reprefentations of the Sides of Objects do vanifh
vanifh upon the Picture, are called by the general Name of Vainifhing Points.

From hence then, we may form an Idea of the Nature of the: Perfpective Plane or Picture, and of Perfpective Reprefentations; which Reprefentations are nothing more than the Section which: the Picture makes with the Rays of Light in their Paffage from. original Objects to our Eyes; and that the whole of this Art, depends upon finding the exact Section, or true Shape, which that cutting of the Rays makes upon the Picture in all kinds of Situations, and in giving them their proper Force and Colour.

But to illuftrate this by a very familiar Inftance. Suppofe a Spectator to be looking at a Profpect without Doors, from within, through a Glafs-Window; he will perceive not only the vaft Extent which fo fmall an Aperture will admit to be feen by his Eye ${ }_{2}$. but the Shape, Size, and Situation, of every Object upon the Glafs: If the Objects are near the Window, the Spaces which they take upon the Glafs will be proportionably larger than when they are at a greater Diftance; if they are parallel to the Window, then their Shapes upon the Glafs will be parallel alfo; but if they are oblique, then their Shapes will be oblique, and fo on. And he will. always perceive, that as he alters the Situation of his Eye, the Situation of the Objects upon the Window will be altered alfo: If he raifes his Eye ever fo high, the Objects will feem to keep pace with his Eye, and rife higher upon the Window; and the contrary, if he places it ever fo low. And fo in every Situation of the Eye, the Objects upon the Window will feem to rife higher or lower; and confequently, the Depth of the whole Profpect will be proportionably greater or lefs, as the Eye is elevated or depreffed; and the Horizon will, in every Situation of the Eye, be upon a Level with it : That is, the Horizontal Line, or that imaginary Line which appears to part the Earth and Sky, will feem to be raifed as far above the Ground upon which the Spectator ftands, as his Eye is removed from the fame Place.
Fig. 43. Let us now fuppofe two Planes $\mathrm{ABab}, \mathrm{CDcd}$, of the fame Height, and parallel to each other, one to pafs through the Eye E, the other through any Point as e , and both to be perpendicular to the Ground ABCD; and let us imagine another Plane, abcd, to be laid upon thefe two Planes, $\mathrm{ABab}, \mathrm{CDcd}$, as in the Figure, and it will be evident, that this Plane abcd is parallel to the Ground $A B C D$, becaufe it lies upon two Planes $A B a b, C D c d$, of the fame Height. Now if we fuppofe this Plane, abcd, to be con-
tinued at an infinite Diftance, and the Line cd to reprefent a Part of the real Horizon, and then imagine a Picture GLPP, to be placed between the Eye E, and the Horizon cd; then its Section HL, with the horizontal Plane abcd, will be the indefinite Reprefentation of the Horizon cd, upon the Picture; and this Reprefentation is called the Horizontal Line. Now fince all Objects which lye flat upon the Ground, or are parallel to it, feem to vanifh into the real Horizon, therefore the Reprefentation of all fuch Objects upon the Picture, muft vanifh into this Horizontal Line ; becaufe it is the perfpective Reprefentation of the real Horizon : And for the fame Reafon, the Ground, or whole Extent between the Eye and the real Horizon, will not appear to lye flat, but to rife upwards. For let E be the Eye, ABCD the Ground, Fig. 44: and HI the utmoft Extent which the Eye can diftinguifh; now, I fay, the Ground will not appear to lie flat, as A BCD, but to rife upwards, like ABcd, 'till it cuts the Plane abcd, which is drawn through the Eye E, parallel to the original Plane $A B C D$; and the Section cd, which the Planes ABcd and abcd make with each other, will reprefent the real Horizon. And, as before, if we fuppofe a Picture, GLPP, to be fixed between the Eye and the faid Horizon ; then the Section HL, which the Hicture makes with the parallel Plane abcd, will be the indefinite Reprefentation of the Horizontal Line upon the Picture; becaufe the Rays of Light, in their Paffage from the Section cd, or real Horizon, would cut the Picture in the Line HL.

From hence then, we may fee, the grand Principle on which Perfpective depends; namely, on finding thofe Lines and Pcints into which Objects feem to vanifh upon the Picture. And whoever will give himfelf the Trouble to underftand the following fhort Theory, will have maftered all the Difficulty in Perfpective : For it only requires to have a clear Idea of the Nature and Property of vanifhing Lines and vanifhing Points, and a few other Requifites as previous thereto; which he may partly conceive by what has been faid already, and by confidering, that as the Horizontal Line HL, is produced by means of the Plane abcd, which paffes through the Eye parallel to the Ground, or original Plane; fo, in the very fame Manner, all other vanifhing Lines are determined; namely, by imagining a Plane to pafs through the Eye, parallel to thofe Planes whofe Reprefentations are required upon the Picture. --Again, in regard to vanifhing Points; they are determined by drawing Lines from the Eye, parallel to the original Lines, 'till
they cut the Ficture; in order to which, we muft always fuppofe thefe Lines to lie in fome Plane, and then, having found the vanifhing Line of that Plane, the vanifhing Point of any Line, in that Plane, may be found alfo. And from hence we nay obferve, that the Horizontal Line is of the fame Nature with any other vanifhing Lines, and differs from them only in being nore ufeful, becaufe, many more Objects are perpendicular and parallel to the Picture, than oblique with it : And therefore, the great 3trefs which hath been laid upon this Line by moft Writers, is not fo very fignificant as they apprehended; for, in fome Cafes, it is of no ufe at all in a Picture. For let us confider a little. If vanifhing Lines upon the Picture, are always to be produced by Planes paffing through the Eye, parallel to original Figures, then no. original Plane can have its vanifhing Line in the Horizontal Line, unlefs it is parallel to the Ground; but, if any Object be obliquely fituated with regard to the Ground, ther, the Plane which is to pafs through the Eye, parallel to the Original, in order to determine its vanifhing Line, will be oblique with the Ground alfo; and therefore it cannot pafs through the Horizontal Line, but will be either above, below, perpendicular to it, or crofs it in an oblique manner: All which may be conceived by infpecting the following Figures. In Fig. 45, the original Object, TOSX, lies upon the Ground; therefore, the Plane, abcd, which paffes through the Eye E, parallel to the Ground, cuts the Picture in the Horizontal Line HL. In Fig. 46, the Original, TOSX, is fuppofed perpendicular to the Ground, and to be perpendicular to the Picture alfo ; therefore, the Plane ABPD, which paffes through the Eye E, parallel to the faid Plane, will be perpendicular to the Ground and perpendicular to the Picture; and therefore will pafs through the Center C of the Picture, and produce the vanifhing Line PD, which will be perpendicular to the Horizontal Line HC. But, if the original Object is perpendicular to the Ground, and oblique with the Picture, as in Fig. 47, then its vanifhing Line PD, will be perpendicular to the Horizontal Line HL, but, will not pafs through the Center or Middle of the Picture, but will be on one Side of it. Again, if the fquare Object A B TS, Fig. 48, (which is inclined to the Ground, at the Angle ATO, but reclined to the Picture) have two Sides AB, TS, parallel to the Picture; then the Plane OPVL, which paffes through the Eye E, parallel to the original ABTS, will produce a vanifhing Line VL, above the Horizontal Line H C, and exactly parallel to it. But if the fame

Object,





Object, (Fig. 50.) be turned fo as to have all its Sides oblique with the Picture, then the Plane EPLV, which paffes through the Eye $E_{3}$ parallel to the original ABTS, will produce a vanifhing Line VL, which will be aflant the Horizontal Line HL. Again; if the Object, ABTS, (Fig. 49.) be inclined both to the Ground and the Picture, but have its Sides AS, BT, parallelito the Picture, (as in Fig. 48.) then its vanifhing Line, VL, will be parallel to the Horizontal Line HL, but below it. And to in regard to the vanifhing Points of any original Lines: As there Lines are fuppofed to lie in fome Planes, therefore, having found the vanifhing Lines of thofe Planes, as above, the vanimhing Point of any Line in thofe Planes may be eafily found alfo; viz. by drawing Lines through the Eye, parallel to fuch Lines, 'till they cut the Picture: Thus, in Fig. 45, EL is drawn from the Eye E, parallel to the Original el, and therefore L is the vanifhing Point of el upon the Picture. And fo again in Fig. 47, Es, EL, and-Eo, are parallel to the Originals $S T, S X, O T$ and $O X$, and therefore will produce the correfponding vanifhing Points; viz. s for the Line ST, L for the Lines SX and OT, and $O$ for the Line XO. In like Manner the Points L, in Fig. 48, 49, 50, are determined; viz. by drawing the Lines EL, from the Eye, parallel to the Originals el and SB.---From hence, then, we may perceive, that the various Situations of Objects may be reduced under three general Heads ; viz.
I. When they are perpendicular to the Picture, or parallel to the Ground.
2. When they are parallel to the Picture, or perpendicular to the Ground.
3. When they are obliquely fituated, both as to the Picture and the Ground, or any other Plane upon which we fuppofe them: All which I fhall now endeavour to explain in their feveral Orders, and apply them to Practice.

## S ECT. If.

## DEFINITIONS

${ }^{4}$ THE Point of Sight, is that Point where the Spectator's Eye is placed to look at the Picture. Thus the Point E, of all the Figures in Plate 6, is the Point of Sight, or Place of the Eye.
Fis. 45 . pendicular to the Picture GLHL, the Point C, where that Line cuts the Picture, is called the Center of the Picture.
3. The Diftance of the PiEfure, is the Length of the Line EC, which is drawn from the Eye, perpendicular to the Picture:
4. If from the Point of Sight E, a Line EP be drawn perpendicular to any vanifhing Line VL, the Point $P$, where that Line cuts the vaniming Line, is called the Center of that vaniJbing Line.
5. The Diftance of a vanifoing Line, is the Length of the Line EP, which is drawn from the Eye perpendicular to the faid Line.
6. By Original Object, is meant the real Object whofe Reprefentation is fought, whether it be a Line, Point, or plane Figure: And by Original Plane, is meant that Plane upon which the
Fig 45. real Object is fituated; thus the Ground OP, is the Original Plane, and TOSX the Original Object.
7. The Line GL, where an original Plane OP cuts the Picture GLHL, is called the Section of the Original Plane, or the Ground Line.
8. If any Original Line OT, be continued fo as to cut the Picture; the Point G, where it cuts the Picture, is called the Interfection of that Original Line.
bin. The Vaidking Line of any Origindl Plane, Ec, is that Line, where a Plane drawn through the Eye, parallel to that Original Plane, cuts the Picture: Thus HL in this Figure, and VL in Fig.48, 49,50 , are the vanifhing Lines of their feveral Original Planes, TOSX and ABTS.
10. The Vanijbing Point of any Original Line, is that Point where a Line drawn from the Eye, parallel to that Original Line, cuts
Fig. 48. the Picture : Thus EL, being parallel to the Original el, produces the vanifhing Point $L$; and fo on.

## Theorem i.

Fig. 5x. If two or more Planes, $\mathrm{ABCD}, \mathrm{EFGH}$, are parallel to each other, they will have the fame vanifhing Line HL.

For let GHLL be the Picture, E the Spectator's Eye, and $A B C D$ an original Object.

Imagine the Plane HIKL to pals through the Eye E, parallel to the original Object $A B C D$, and it will cut the Picture in the Line HL, which will be the vanifhing Line of the original Plane ABCD: And fince the other original Plane EFGH, is parallel to ABCD, therefore the Plane HIKL is parallel to that alfo; and confequently HL is the vanifhing Line of the Plane EFGH, and of every other Plane which is parallel to $A B C D$.

## THEOREM 2.

The vanifhing Points, H and L ; of Lines $\mathrm{AC}, \mathrm{BD}$, in any original Plane ABCD, are in the vanifhing Line HL, of that Plane.

From the Eye E, draw EH, EL, parallel to BD and AC; then becaufe the original Plane ABCD, and the Plane HIKL, are parallel; therefore the Lines EH, EL, that are drawn from the Eye E, parallel to the original Lines $\mathrm{BD}, \mathrm{AC}$, will be in the Plane HIKL; and confequently muft cut the Horizontal, or vanifhing Line HL, in the Points $\mathrm{H}, \mathrm{L}$, and thereby produce the proper vanifhing Points of the original Lines BD, AC.

$$
\text { THEOREM } 3 \text {. }
$$

If the original Plane $A B C D$, is parallel to the Picture GHLL, Fig. $5^{2}$. it can have no vanifhing Line upon it, and therefore its Reprefentation will be parallel, as in Fig. 42. becaufe its parallel Plane abcd, which paffes through the Eye E, can never cut the Picture, and confequently, will not produce a vanihhing line upon it. And fo in regard to the Line BD : It can have no vanihing Point upon the Picture, but its Reprefentation will be parallel to the Original, as os, $t x$, in the above Figure.

## THEOREM 4 .

The Reprefentation $a b$, of a Line $A B$, is a Part of the Line Fig. 53 , GC, which paffes through the interfecting Point $G$, and the va ${ }^{-1}$ nifhing Point $C$, of the original Line $A B$.

For imagine the Plane AHEF, to pafs through the Eye E, and the original Line $A B$, and it will pafs through both the interfecting Point $G$ and the vanifhing Point $C$, and cut the Picture in the Line GC: And if the vifual Rays AE, BE, are drawn from the Object to the Eye, they muft be in the Plane AHEF,

$$
\text { E } 2 \text { and }
$$

and confequently, their Section $\mathrm{a} b$ with the Picture, will be in the Section GC of that Plane with the Picture; therefore, $a_{2} b_{2}$ which is a Part of the Line GC, is the Reprefentation of the Line AB.

## COROL. I.

When the Original is perpendicular, as AB, Fig: 53, then its vanifhing Point will be in the Center C of the Picture; becaufe a Line drawn from the Eye perpendicular to the Picture, determines its Center; and therefore, fince $A B$ is perpendicular to the Picture, EC is parallel to it, and confequently will produce the Center $\mathbb{C}$, for the vanilhing Point of $A B$.

$$
\text { COROL. } 2 .
$$

If the Original AB is in a Plane $O P B$, perpendicular to the Picture, but lies obliquely in that Plane in regard to the Picture, Fig. 54. as AB; then its vanifhing Point $L$, will be in the Horizontal Line. HL, But on one Side of the Center C: And fo whatever be the $\mathrm{Si}-$ tuation of any original Line, its Reprefentation upon the Picture will always be in that Line which is drawn through its Interfection and vanifhing Point.

$$
\text { COROI. } 3
$$

Fig. 55. For let $A B$ be inclined to the original Plane $O P$, at the Angle ABD.

Continue $A B$ 'till it cuts the Picture in $G$, and from the Eye $E_{2}$ draw EF parallel to it, which will cut the Picture in the vanifhing Point F; then draw FG, and the vifual Rays AE, BE, cutting FG , in a and b ; then will the Line ab be the Reprefentation of the Original $A B$, and is a Part of the Line $\mathrm{FG}_{2}$ which pafles through the interfecting Point $G$, and the vanifhing Point $F$, of the Original AB. This, from what was obferved above, is felfevident; becaufe the Rays AE, BE, are in the Plane AFEG, which paffes through the Eye and the original Object, and therefore muft cut the Pieture in the Section $F G_{\text {. }}$.

## COROL. 4.

From hence it follows, that all Lines which are parallel to each other, but not parallel to the Picture, will have the fame vanifhing Point; becaufé a Line which paffes through the Eye, being parallel to one, is parallel to all the reft; and therefore can produce bart
VII.


one vaniming Point, let the Number of parallel Lines be ever fo many. This I have explained by Paper Planes, where OPHL is Fig. 56. the Picture, TPE a Plane which paffes through the Eye parallel to the Picture, and $A B, C D, E F$, three original Lines parallel to each other. Now if we raife the Picture OPHL, and the Plane TPE, 'till they are perpendicular to the original Plane AEKI, and then turn the other Planes, which pafs through the original Objects $\mathrm{AB}, \mathrm{CD}, \mathrm{EF}$, 'till they coincide with the Eye at E ; they will all meet upon the Picture in the Point $C$, which is the common vanifhing Point of all the original Lines $\mathrm{AB}, \mathrm{CD}$ and EF . And by obferving the vifual Rays, which are drawn from the Extremities of every original Object to the Eye, at E, we may perceive that the Reprefentation of the Line $A B$, will be $a b$ upon the Picture; of CD, cd; and of EF, ef: All which Reprefentations will tend to the Point $C$, as a common Center, and there vanifh into the Picture. And we may moreover obferve, that fince the original Lines $\mathrm{AB}, \mathrm{CD}, \mathrm{EF}$, are not only equal and parallel to each other, but at equal Diftances from the Picture or Section GL; that therefore their Reprefentations will be at the fame Diftance from the Section, GL, of the original Plane, and between the fame parallel Lines a e, bf.

This laft Theorem, and the Corollaries deduced from it, are the principal Foundation of all the Practice of Perfpective; and therefore the Reader will do well to make it very familiar to him: And to help his Reflections upon it, I have annexed the laft Figure. But although I have confined myfelf in this Figure to an original Plane which is perpendicular to the Picture; yet the fame Rules will ferve for any other original Planes, be they ever fo obliquely fituated in regard to the Picture; provided they are parallel amongft themfelves: As muft appear extremely obvious, by a little Attention in examining the Figure.

$$
\text { THEOREM } 5 \text {. }
$$

The Reprefentation ab, of any Line AB, that is parallel to the Fig. ${ }_{5} 6$. Picture, is to its original Line $A B$, as the Diftance EC of the No. 2 . Reprefentation $a b$ is to the Diftance ED of the original Figure. For let the original Figure AB be two Parts, and the Diftance ED (or which is the fame Thing, AH) five Parts; and the Diftance EC, (or HG) of the Reprefentation ab, two Parts; then will $A B$ be to its Diftance $E D$ as five to two. For if we divide
the Diftance $C E$ of the Reprefentation $a b$, into five Parts, then the Reprefentation ab will be equal to two of thofe Parts; that is, as five is to two. Again, the Diftance Ca , between the vanifhing Point C, of a Line AO, and any Point a in its Reprefentation Oa ; is to the Diftance CO , between the vanifhing Point C and the Interfection of that Line, as the Diftance EC (or HO) of the Eye, is to the Diftance HA of the original Point. For let HA be five Parts, and HO two Parts; divide OC into five Parts; and the Diftance Ca , between the Reprefentation a of the Point A , will be two of thofe Parts; therefore, Ca is to CO , as HO is to HA; that is, as two is to five: As is evident by infpecting the Figure.

From hence, then, we may obferve, that the perfpective Reprefentations of Objects are diminifhed upon the Picture in an harmonical Proportion; and that, if the Length of any original Object, its Diftance, together with the Diftance and Height of the Eye, are known, that then the Appearance of thofe Objects upon the Picture may be found by Calculation; which will be exemplified in the practical Part. Proceed we, therefore, in our propofed Order*, to determine the Reprefentations of Objects which are in Planes varioully fituated in regard to the Picture.

## S E C T. III.

Of $\mathrm{OBjECTS}_{\mathrm{s}}$ which are in Planes perperdicular to the Picture. +
Fig. 57.
$\mathrm{Na}_{1}{ }^{2}$.

L$E T A B C D$ be a qquare Object lying flat on the original From one From what has been faid already it is manifeft, that $a b c d$ is the Reprefentation of $A B C D$; for the Points $a, b, c, d$, are where the vifual Rays $\mathrm{BE}, \mathrm{O}_{\mathrm{C}}$. are cut by the Picture, as was oblerved in Fig.41, 42. Or the Reprefentations ab, cd, are Parts of the Lines TC, SC, which are drawn from the interfecting Points T and S, and the vanifhing Point $C$, of the original Lines $A B, C D$; as was thewn in Fig. 53, 54; and confequently ad, bc, are the Reprefentations of their Originals AD, BC.

[^12]
## Of Perpendicular Objects.

Now let us fuppofe the original Plane: OGLP to be turned upon its Section GL; and the parallel Plane HIKL to be turned alfo upon the vanifhing Line HL, 'till thofe Planes and the Picture
 Eye E, will be tranfpofed into the Point $\mathbb{E}$, and $\mathbb{C} C$ will be equal to its Diftance. And if we moreover fuppofe the original Figure $A B C D$, to be drawn upon the under Side of the Plane OGLP, and exactly in the fame Situation as $\mathfrak{A D C} \mathbb{C}$ in the Plane $\mathbb{D G L} 19$; then, I fay, if Lines are drawn from the feveral Points $\operatorname{GLB} \mathbb{C} D$ in this tranfpofed Plane, to $\mathbb{E}$ the tranfpofed Place of the Eye, that their Sections a, b, c, d, with the Lines TC, SC, will be in the very fame Points, in which thofe Lines are cut by the Rays, which go from the original Points A, B, C,D, in the Plane OGLP, to the Eye E: Thus the Ray BE cuts the Line TC in b; and if a Line is drawn from 25 to $\mathbb{E}$, it will cut TC in the fame Point $b$; and fo of the reft. From whence it follows, that the Reprefentation abcd, may be as exactly determined by thus tranfpofing the Planes, as by thofe imaginary Rays of Light which go from the real Object to the Eye.

That the Senfe of this Figure may be the more clearly compreliended, in Fig. 57, No. 2, are all the above Planes laid flat upon the Paper; and may eafily be diftinguifhed by the Letters which denominate each Plane. Thus OPLG is the original Plane, ABCD the original Object, $T$ and $S$ the Section of the Sides $A B, C D$, with the Picture GLHL: The parallel Plane is HIKL; and HL the vanifhing Line of the original Object. C the Center of the Picture; E the Eye; and EC its Diftance...-Thefe Things being premifed, let us apply them to Practice by drawing the above Reprefentation.

From T and S draw TC, SC, and from the feveral Points Fig. ${ }^{57}$ $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, draw Lines to the Eye at E , which will cut $\mathrm{TC}, \mathrm{SC}$, No. 2 . in the Points $a, b, c, d$; then draw ad, bc, parallel to HL , and the Reprefentation is compleated.

From hence, then, it follows, that if the Situation, or Seat of an original Object; together with the Place of the Picture, and the Diftance of the Eye, are known, that then the Reprefentation of that Object may be eafily determined: For let us now, without any. Regard to the former Figure, call OPGL the Ground, ABCD an original Object, GLHL the Picture, HL the Horizontal Line, C the Center of the Picture; and CE the Diftance of the Eye.

From the Eye E draw EC, parallel to the Sides $A B, C D$ of the Original, which will cut the vanifhing Line HL, in $C$, the Center of the Picture ; becaufe AB and CD are perpendicular to the Pioture, that is, perpendicular to the Section GI; therefore C is the vanifhing Point of AB and $\mathrm{CD} .--$ Continue the Sides AB , CD, 'till they cut the Section GL in T and S. From T and S draw Lines to C ; then from the feveral Points $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, draw Lines to E, which will cut TC, SC, in the Points a, b, c, d: Finally, draw the Lines ad, b c, which will give the Reprefentation required.

This Reprefentation may alfo be determined without drawing Lines from the original Points A, B, C, D, to the Eye E, by means of the Diagonal AC continued, and its parallel E N.----For Continue the vanifhing Line HL, and the Section GL, at pleafure; continue alfo the Diagonal AC, 'till it cuts the Section in $M$ : From E, draw EN, parallel to $A C$; and from N, where EN cuts the vanifhing Line, draw NM, cutting TC, SC, in the Points a and $c$; then is a the Reprefentation of $A$, and $c$ the Reprefentation of $C$; therefore from a and $c$, draw $a d, b c$, parallel to HL, and the Thing propofed is done.
Fig. 58. For let $A B C D$ be an original Square, and AC, BD, Diagonals drawn in it; and let ABcd be its Reprefentation upon the Pic-ture.--C is the Center of the Picture, and CE its Diftance.

Through E, draw EL and EH, parallel to the Diagonals AC, $B D$, cutting the vanifhing Line in $L$ and $H$; then are $L$ and $H$ the vanifhing Points of thofe Diagonals; for there the Picture is cut by Lines which are drawn from the Eye parallel to the Originals AC, BD. And for the fame Reafon, (as we have obferved before) C is the vanifhing Point of $\mathrm{AD}, \mathrm{BC}$; and therefore, if Lines are drawn from the Sections A, B, to the vanifhing Points H, C, L, their mutual Interfections c, d, with AC, and BC, will determine their feveral Reprefentations: Thus Ad is the Reprefentation of $A D, B c$ of $B C, A c$ of $A C$, and $B d$ of $B D$; and by drawing cd (which will be parallel to the Horizontal Line) the Reprefentation of the whole Square will be compleated.

The practical Part is reprefented by the 59th Figure; where all the Planes are laid down, as before, with correfponding Letters to diftinguifh them.

From hence, then, we may obferve, that any plane Figure may eafily be drawn upon the Picture by refolving the whole into Triangles.




## Of PERPENDTCULAR OBJECTS.

For let ABCD be a Square refolved into four Triangles, as Fig 59 . $A N D, A N B, C N D, C N B$. Then, by means of the three vanifhing Points $\mathrm{H}, \mathrm{C}, \mathrm{L}$, which are found by drawing EC, EH, EL, parallel to $A D, B C, A C, B D$, the Reprefentations of thofe Triangles may be found; as in the Figure. And fo likewife in Fig. 60, the Reprefentation of the Parallelogram ABCD, by means of the Points $\mathrm{H}, \mathrm{L}$; or the Triangles $\mathrm{ABC}, \mathrm{ADC}$, by means of the Points P, H, L, may be determined.

Thefe two laft Figures, though fo very fimple, contain the greateft Part of Practical Perfpective: For, however original Planes are fituated, or however any Lines are drawn upon them, their Reprefentations may always be determined upon the Picture, by continuing the original Lines 'till they cut the Picture, and by drawing Lines through the Eye parallel to them. All the Difficulty lies in being careful to: draw the Lines from the right. interfecting and vanifhing Points; which a little Practice will make extremely eafy: And, therefore, here the Learner will do well to exercife himfelf with the Examples under this Head in Book II. Sect. 2.

## S E C T. IV.

Of OBJECTs wbich are in Planes perpendicular to the Ground.

HEre $\operatorname{TOSX}$ is a fquare Plane which flands upon its Side TO, Fig. 6r, perpendicular to the Ground Plane OP, and is alfo perpendicular to the Picture.

Now let $\mathbb{E}$ be the Eye, C the Center of the Picture, and CE its Diftance.--From the Eye E, draw EH parallel to TX or OS, and EC parallel to OT: Then becaufe EC is parallel to TO and SX , therefore C is the vanifhing Point of thofe Lines; and therefore, from C draw CL, cutting the Section GL in L; and then from L draw LH parallel to CE, which will compleat a perpendicular Plane CEHL, that paffes through the Eye paralleb to the original Cbject TOSX; and therefore CL, its Section with the Picture, is the vanifhing Line of that original Plane. And fince CE is by Conftruction perpendicular to CL , therefore C is: the Center of the vanifhing Line, and alfo the Center of the Pic-: ture, and CE is its Diftance.

Again, continue CL at pleafure; and from the Eye E, draw EA, EB , parallel to the Diagonals $\mathrm{OX}, \mathrm{T}$, , which will cut the vanining,

Line $A B$ in the Points $A$ and $B$; therefore $A$ and $B$ are the vanikhing Points of thofe Diagonals, by means of which the whole Reprefentation may be determined. Thus $G$ is the Section of the Side OT, and C its vanifhing Point, therefore draw GC; then from T and O draw Lines to E , which will give the Ap pearance to of TO; and from $t$ and o draw the Lines $t x, 0, s$, parallel to the vanifhing Line $A B$ (that is, perpendicular to the Ground Plane) and continue them at pleafure: Finally, from A draw a Line through $o$, cutting $t x$ in $x$, and from $B$ draw a Line to $t$, which will cut os in $s$, then draw $s x$ to its vanifhing Point $C$, which finifhes the Figure.
Fig. 62. But to apply this to Practice.---The Planes being fuppofed to be laid flat, as in Fig. 57. No. 2.

Then OT reprefents the Seat, or Plan, of the original PlaneTOSX, in the laft Figure, TG its Diftance from the Picture, AEB the parallel Plane, E the tranfpofed Place of the Eye, and CE its Diftance.

From the Extremities O, T, of the Seat OT, draw $\mathrm{T}_{1}, \mathrm{O}_{2}$, at pleafure, but parallel to each ether, cutting the Section in I and 2; make CB equal to the Diftance CE of the Picture, and from B draw BH , parallel to T , O 2; cutting the horizontal Line in H : Then is $\mathrm{H}_{\text {the }}$ thenifhing Point of the Lines $\mathrm{T}_{1}, \mathrm{O}_{2}$; therefore draw $\mathrm{H}_{1}, \mathrm{H}_{2}$, and from G draw GC, which will be cut by the above Lines in the Points $t$, $o$; and thereby give to for the Reprefentation of TO. Again, from $t$ and $\odot$, draw the Lines $t x$, os, at pleafure, but parallel to the vanifhing Line $A B$; then from A draw a Line through $o$, cutting $t x$ in $x$; and from $B$ draw a Line to $t$, which cutting os in $s$, will determine the laft Angle of the Square; and therefore, by drawing $s \times$ to its vanifhing Point C, the whole Reprefentation will be compleated.-I have made ufe of both the vanifhing Points A, B, to exercife the Learner, but one Point will do; thus, Ax determines the Side $t \mathrm{x}$; therefore draw x.C, which will cut os, and give the other side os:
Fig. 61. Here let us obferve, that when the Seat OT, of any Plane, is perpendicular to the Picture, the vanifhing Line of that Plane will pafs through the Center of the Picture, and be perpendicular to the horizontal Line: But, if: the Seat OT, Fig. 63 , of any perpendicular Plane, TOSX, be oblique with the Picture, then its: vanifhing Line, $A B$, will not pafs through the Center of the Picture, but on one Side of it; neverthelefs, it will always be perpendicular to the horizontal Line, and will pats through the vanifhing Point L , of its Seat OT.

For

## of Parallel Objects.

For, draw EL, parallel to OT, and it will cut the horizontal Fig. 63. Line in L: From E and L, draw EH, LL, parallel to TX or OS; and from L, where LL cuts the Section GL, draw LH parallel to EL; then is the Plane LLHE parallel to the original Plane TOSX, and confequently perpendicular to the Ground; and therefore LL, its Section with the Picture, is the vanifhing Line of that original Plane, and is perpendicular to the horizontal Line: And fince the vanifhing Point L is in the Section LL, therefore LL continued will pafs through that Point, and confequently $A B$ is the vanifhing Line of the Plane TOSX. Again; fince $E L$ is perpendicular to the vanifhing Line $A B$, therefore $L$ is the Center of that vanifhing Line, and EL its Diftance; and therefore, from E draw EA, parallel to the Diagonal OX, and EB parallel to the Diagonal TS, cutting the vanifhing Line in A and $B$; then are $A$ and $B$ the vanifhing Points of thofe Diagonals; from whence the Reprefentation may be compleated, as in the former Figure.

But to apply this to Practice. Let the feveral Planes be fuppofed to be laid down as before.

Then TO is the Seat of the original Object, $L$ its vanifhing Fig. 64: Point, $C$ the Center of the Picture, EC its Diftance, L the Center of the vaniming Line $A B$, and EL its Diftance.

From the Section G, draw GL to its vanifhing Point, and from the Extremities T, O, of the Seat TO, draw two parallel Lines at pleafure, cutting the Section GL in 1 and 2 ; from $E$, draw $\mathrm{E}_{3}$, parallel to $\mathrm{T}_{1}$ and $\mathrm{O}_{2}$, cutting the horizontal Line in 3; then draw 13, 23 , which will give the Reprefentation ot ; again, from t and o , draw the Lines tx , os, parallel to the vanifhing Line AB: And then, by means of the vanifhing Points A and B, the whole Reprefentation may be compleated, as in Fig. 62.

This Figure alfo deferves the Learner's particular Attention; for if he obferves, in Fig. 62, the vanifhing Line AB paffes through the Center of the Picture, and therefore the Diftance CE of that vanifhing Line, is equal to the Diftance of the Eye, or principal Diftance: But in this laft Figure, fince the vanifhing Line does not pafs through the Center of the Picture, therefore, the Diftance EL, of that vanifhing Line, is greater than the principal Diftance CE , and will be proportionably greater and greater, as the vanifhing Line is removed farther and farther from the Center of the Picture. For the principal Diftance EC, is one Side of a right- Fig. 690 angle Triangle ECL ; but EL, the Diftance of the vanifhing Line
$A B$, is the Hypothenuife of that Angle, and therefore greater than either of the Sides EC or C L : From whence it follows, that if a Line CL be drawn from the Center of the Picture, perpendicular to any vanifhing Line AB, the Point $L$, where that Line cuts the Picture, will determine the Center of that vanifhing Line; and if a Line be drawn from the Eye to that Point, as E L, it will determine its Diftance *.

Let us now, without any Regard to the Theory, find the Appearance of a fquare Object fituated like TOSX, in Fig. 63.
Wug. 64. Let TO be the Seat of the Objeet propofed, HL the horizontal Line, $C$ the Center of the Picture, CE the principal Diftance, and GL the Section of the Ground Plane with the Picture. Continue the Seat OT, 'till it cuts the Section in G, and parallel to OT, draw EL from the Eye, cutting the horizontal Line in the vanifhing Point L ; then draw GL : Finally, draw $\mathrm{T}_{1}, \mathrm{O}_{2}$, and and alfo their Parallel $\mathrm{E}_{3}$; by which means the Reprefentation ot $t_{\text {, }}$ may be found. Again, through the vanifhing Point $L$, draw $\mathrm{BA}_{2}$ perpendicular to HL, and continue the horizontal Line towards $\mathbb{E}$, at pleafure ; then, becaufe CL is perpendicular to the vanifhing Line $A B$, therefore $L$ is the Center of that vanifhing Line, and confequently, EL is its Diftance: Therefore continue the Perpendicular CL, at pleafure, beyond the vanifhing Line AB, and from L, with the Radius LE, defcribe an Arc $A \mathbb{C} B E$, cutting the vanifhing Line in $A$ and $B$, and $C L$ continued in $\mathbb{C}$; then are $A$ and $B$ the vanifhing Points of the Diagonals $o x$, and $t s$, and $\mathbb{E}$ is the proper Diftance of the Eye : Therefore by drawing Perpendiculars from $t$ and $o$, and Lines from $A$ and $B$, through the fame Points $t$ and $o$, they will cut the Perpendiculars $t x, 0 s$, in $x$ and $s$, and thereby give the Height of the Square; from whence, by drawing $x L$, it will be compleated.

From hence, then, it is manifeft, that the Method for finding the Reprefentation of an upright Plane, is exactly the fame as that for determining the Appearance of a Plane which lies flat upon the Ground, only the Situation of the vanifhing Line is different; but the Operation in both Cafes is the very fame; which may be conceived by turning the Figure, and imagining AB to be the horizontal Line, L the Center, and L© the Dittance of the Picture : For then this Figure will be like Fig. 59. But that the Learner.

[^13]
may underftand the Meaning of this more perfectly, he is defired, before he proceeds any farther, to exercife himfelf with fome Examples of this Kind; which he will find in Book II. Sect 3.

## S E C T. V.

Of OBJECTS which are inclined to the Ground.

'HE Objects which come next under Confideration, are fuch as are neither perpendicular nor parallel to the Ground, but inclined to it; like the Roofs of Houfes, Pediments; and the like; the vanifing Lines of which cannot be the horizontal Line, nor any Line that is perpendicular to it.

For let TOSXYZ be the original Object, having one Side Fig. 6 . TOSX upon the Ground, and one side TOYZ inclined to it, at the Angle Y T X; and let the other Sides be perpendicular to the Ground...-E is the Eye, C the Center of the Picture, and CE its Diftance.
From E draw EC, parallel to XT or SO; then is C their vanifhing Point: And becaufe TX is perpendicular to the Picture, therefore its vanifhing Point C is the Center of the Picture. And fince the Plane TXY, is perpendicular to the Ground, therefore its vanißhing Line LD, is perpendicular to the horizontal Line; and therefore, through the vanifing Point C , draw LD, which continue at pleafure, then from $E$ draw $E D$, parallel to $T Y$ or $O Z$, which will cut the vanifhing Line LD, and give D for the vanifiing Point of the inclined Sides TY, OZ. And if a Line, VL, be drawn through $D$, parallel to the horizontal Line HC, it will be the vanifhing Line of the inclined Plane, TYZO; becaufe, if a Plane was to pafs through the Eye, parallel to TYZO, it would cut the Picture in the Line VL. And fince ED is perpendicular to the vanifhing Line VL, therefore D is the Center of that vanifhing Line, and ED its Diftance.

To apply this to Practice. Let us fuppofe the Planes to be laid down as in the former Figures; only for Convenience, we have removed the Seat TOSX, farther from the Middle of the Picture. ----Here TOSX is the Seat of the original Object, H L the hori- Fig, 66. zontal Line, E the Eye, C the Center of the Picture, and CE its Diftance.

Find the Reprefentation of TOSX, as before directed, by means' of the Lines $\mathrm{O}_{\mathrm{I}}, \mathrm{S} .2$, and their parallel DH : Then, parallel to
the horizontal Line HE, draw a Line ab, Fig. $z$, at pleafure, and through C draw the vanifhing Line DL, perpendicular to the horizontal Line, at pleafure alfo : With the Line ab , and at the Point b, make an Angle abc, equal to XTY, the Angle of Incli-
*Fig. 65. nation of the original Figure ; * then from E, the Diftance of the Eye or principal Diftance, draw ED parallel to bc, cutting the vanifhing Line in $D$; finally, from $D$ draw $D t, D o$, and from $s$ draw $s z$, parallel to $D L$, which will cut $\circ D$ in $z$; therefore ${ }_{z}$ from z, draw zy parallel to $t \mathrm{O}$, or HE, and the Thing propoled is done.

Or the vanifhing Point $D$ may be determined without the Figure $Z_{2}$ by making an Angle at $E$, the Diftance of the Eye, with the horizontal Line HE , equal to the Angle of Inclination, and then drawing ED.

In Fig. 65, the Plane TOZY is inclined to the Ground Plane, but reclined in refpect to the Picture, and therefore its vanifhing Line VL will be above the horizontal Line: But in Fig. 67, the inclined Plane. T OZY is inclined to the Ground and to the Picture alfo; for which Reafon, its vanifhing Line VD will be below the horizontal Line.---The 68th Fig. reprefents the laft Figure applied to Practice, the Operations of which are the very fame with thofe in Fig. 66; only the Seat TOSX, and the vaniming Point D, are inverted; that is, are below, inftead of above the horizontal Line.
Fig. 65 , From hence, then, it is evident, that D is the vaniming Point ${ }_{68} 86,67, \&$ of all Lines which are parallel to the Sides $o z$, and $t y$; and therefore, when the Figure confifts only of parallel Sides, as oz. and $t y$, there will be no Occafion for drawing the vanifhing Fig. 65. Line VL or VD; fince the vanifhing Point D of thofe Sides

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-67
$$ is only wanted. But if any other Lines are fuppofed to be drawn upon the inclined Plane, as in Fig. 69, then thofe vanifhing Lines become neceffary; becaufe the vanifhing Points of thofe Lines will be fomewhere in them. Which comes next under Confideration.

Fig. 6g. Let tozy be the Reprefentation of one inclined Plane, whofe vanifhing Point is $D$; and cdef another inclined Plane, whofe vanifhing Point is D; and let VDL be their vanifhing Lines. ---E is fuppofed the Eye, C the Center of the Picture, and CE its Diftance.--Continue the vanifhing Line DD at pleafure : Then, becaufe CD is drawn from the Center of the Picture, perpendicular to the vaniming Lines VL, VL, therefore $\mathrm{D}, \mathrm{D}$, are the Centers of thofe vanihing Lines, and $\mathrm{DE}, \mathrm{DE}$, theirDiftance from the
the Eye; confequently if DI, DI, be made equal to DE, DE, then I, I, will reprefent the tranfpofed Places of the Eye; and therefore if Lines are drawn from the Points I, I, parallel to any original Lines, they will cut the vanifhing Lines VL, VL, and give the vanifhing Points of fuch Lines. Thus, let it be required to find the vanifhing Points of the Diagonals of a Square, to 12, one of whofe Sides to is given.---Any where apart draw a Square, as X , at pleafure, but in fuch a Manner that its Sides, ab, cd, are parallel to the vanifhing Line V L; and likewife draw its Diagonals. ---Firft for the Figure toszy.

From I, draw IL, IV, parallel to ac, bd; which will cut the vanifhing Line in $V$ and $L$; and from $t$ draw $t L$, cutting $\circ D$ in 2; from 0 , draw oV, cutting tD in 1 ; then draw 12 parallel with $t 0$, and then is $t 012$ the Reprefentation of a Square upon the inclined Plane tozy; and $t 2,0 \mathrm{I}$, are the Reprefentations of its Diagonals. And were it demanded to make the Length of the inclined Plane equal to feveral Times its Width, as in this Figure, we may do it by means of the Points $V$ and $L$; becaufe having determined one Square, all the reft are to be found in the fame Manner.

Here let us take Notice, that if one vanifhing Point of any Plane is determined, all the other vanifhing Points of Lines which can be drawn any how in that Plane, will be fomewhere in a Line which is drawn through that Point. Thus C is the vanifhing Point of the Side 0 s, which lies upon the Ground, and the horizontal Line HE paffes through that Point: Again, $\mathcal{C}$ is the vanifhing Point of o s', which is' one Side of the perpendicular Plane osz; therefore DCD, the vanifhing Line of that perpendicular Plane, paffes through the Point C: And fo again, D is the vanifhing Point of the inclined Planes, and therefore VL, VL, their feveral vaniffing Lines, will pafs through the Points D, D; and confequently, all the Lines, which can be drawn in either Plane, will have their vaniffing Points fomewhere in the vanifhing Lines of thofe Planes. All which is explained by various Examples in the fecond' Book.

Hitherto I have confidered the inclined Planes, as having one or more of their Sides parallel to the Picture, for which Reafon the vanifhing Lines of thofe Planes are parallel to the horizontal Line. Let us now fuppofe the Plane to be fituated in fuch a Manner as to have all its Sides oblique with the Picture, as in Fig. 70.

Fig. 7o. Here TOZY, is a fquare Plane every way oblique with the Picture ; TOSX, its Seat on the Ground; YTX, its Angle of Inclination; E the Eye; C the Center of the Picture, and CE its Diftance.--Draw the Horizontal Line HC, and continue it at pleafure; then parallel to TX, or OS, draw EH, cutting the Horizontal Line in H ; and then is H the vanifhing Point of the Lines TX, OS. Again, parallel to TO, or SX, draw EL, cutting the Horizontal Line in L ; then is L the vanifhing Point of the Lines TO, S X; from whence the Reprefentation of its Seat may be found. Now fince the Plane TYX is perpendicular to the Ground, its vanifhing, Line HV will be perpendicular to the Horizontal Line; therefore from the vanifhing Point H, draw HV parallel to XY, and EV parallel to TY, cuitting HV in V; then is $V$ the vanifhing Point of the parallel Sides TY, OZ; and fince L is the vanifhing Point of TO, it is alfo the vanifhing Point of its parallel Side $Y Z$, and therefore, a Line drawn through $V$ and L , will be the vanifhing Line, (as VL) of the inclined Plane TOZY. Here let us obferve again, that if a Line, ED, be drawn from the Eye E, perpendicular to the vanifhing Line VL, then D is its Center, and DE its Diftance.

To apply this to Practice.--Imagine the feveral Planes to be laid down as before.
7ig. 9 .
Then, HL is the Horizontal Line, E the Eye, C the Center of the Picture, CE its Diftance, HV the vanifhing Line of the perpendicular Plane tyx;.VL, the vanifhing Line of the oblique Plane tozy, $\mathbb{C}$ its: Center, $\mathbb{C} \mathbb{E}$ its Diftance, and $H, L, V$, the vanifhing Points of the feveral Planes; or, if you pleafe, of the feveral sides of fuch a Figure.

Let ot be given for the nearef. Side. Continue ot, 'till it cuts the vanifhing Line HL in its proper vanifhing Point L: From L draw LCE, and from $t$ and $o$, draw Lines, to the vanifhing Point $V$, and draw VCE: Then, is V退L a right Angle; which bifect, and
 vanifhing Point of the Diagonal of a Square: Therefore (fince the inclined Plane was fuppofed to be a Square) draw $\Phi t$, cutting $0 V$ in $z$; from $L$, through the Point $z$, draw Lzy, cutting $t V$ in $y$; then draw $y z$, parallel to $H V$, which will compleat the whole Reprefentation, not only of the inclined fquare Plane, but the whole Appearance of a Figure like 65, 67, but in a different Situation.

## Of Inclined Objects.

Since this Figure is as difficult in regard to the Practice of Per${ }_{\text {Ip }}$ pective, as any I can think of, I have annexed the Paper Planes in the 72 d Figure, to help the Reader's Reflections upon it ; and to affift him itill further, we will now find the Reprefentation of fuch an Object without any Regard to the Theory.

Let E be the Eye, C the Center of the Picture, CE its Dif- Fig. ${ }^{71}$. tance, HL the horizontal Line, and to one Side given of the inclined Face.

Any where apart draw AB, Fig. X. parallel to the horizontal Line HL, and draw $C B$ perpendicular to $A B$; then make an Angle at A, equal to the Angle of Inclination (as TYX in Fig. 70) and draw AC.-.Continue ot to its vanifhing Point L, and from L draw LE to the Eye; then at E make a right Angle with the Line LE, and then, becaufe the Side which lies upon the Ground is fquare at the Corners, therefore H is the vanifhing Point of the two Sides $t x$ and os, and $L$ is the vanifhing Point of the other two Sides to and $s$ x.---From the vanifhing Point H, draw HV perpendicular to the horizontal Line, and continue the horizontal Line towards 7. From H fet off H3, equal to the Diftance HE of the vanihing Line HV ; then from 3 draw 3 V , parallel to AC in Fig. $X$; which will cut HV in V, and give HV for the vanifhing Line of the perpendicular Plane tyx; and by drawing a Line through the Points V and L , we fhall have VL for the vanifhing Line of the inclined Plane tozy: Therefore from C the Center of the Picture, draw $\mathrm{C} \mathbb{C}$, perpendicular to the vanifhing Line VL, and continue it at pleafure; then is $\mathbb{T}$ the Center of that vanifhing Line. Again, from C the Center of the Picture, draw CI perpendicular to $C \mathbb{I}$, and make CI equal to CE the principal Diftance, and then draw $I \mathbb{T}$, which is the Diftance of the vanihing Line VL; therefore, make $\mathbb{C} \mathbb{\mathbb { E }}$ equal to $\mathbb{C} 1$, and from the vanifhing Points $V$ and $L$, draw V $\mathbb{E}, L \mathbb{E}$, which will be a right Angle: Bifect the Angle $\mathbb{C}$, and draw $\mathbb{C} \mathbb{T}$, cutting the vanifhing Line in $\$$; then, as before, $\ddagger$ is the vanifhing Point of the Diagonal of a Square tozy, from whence the whole Reprefentation may be compleated. Here alfo the Learner is referred for Examples to Book II. Chap. 2. Sect. 4.

Thus have I endeavoured to explain the Theory of Pe:fpective, and to apply it to Practice by the moft familiar and ufeful Examples, and in all the Variety of Inftances which can come within the general Practice of Painting, \&c. As for other Matters, which are out of the common Road, and which ferve rather to
perplex than benefit a Learner, I have purpofely avoided them: and believe, I may venture to affirm, that whoever has attended. to what has been faid, and exercifed himfelf regularly with the Examples to which he was referred in the Practical Part, Book the Second, will find no kind of Difficulty in determining the Appearances of any Objects upon an upright Picture, let them be of ever. fo irregular a Figure, or howfoever they are fituated

But thus far I have confined myfelf to the Appearance of Ob jects upon an upright Picture only, fuch as are generally made choice of for Perfpective Reprefentations: But as there are fome Cafes in which the Situation of the Picture is different, fuch as Ceilings, inclined Walls, or the like, I thall now proceed to the Confideration thereof, and fhew, that the Reprefentation of Objects upon fuch kind of Surfaces, is deducible from the fame Principles, and confequently, is to be determined after the fame Manner; which is the Subject of the next Chapter.




## C H A P. IV.

## Of Parallel and Inclined Pictureso

## SECT. I.

Of the Parallel Picture

WHEN the Picture is perpendicular to the Ground, or any other Plane upon which the Spectator ftands, I call it a perpendicular Picture; when it is parallel to the Ground, I call it a parallel Picture; and when it is inclined to the Ground, I call it an inclined Picture. The firft of thefe Situations I have already confidered at large, as being the moft ufeful: Proceed we therefore to the Second, which principally relates to Ceilings or immoveable Pictures.

Now, whoever has attended to what hath been faid upon the upright Picture, will (I apprehend) find no fort of Intricacy in this, becaufe, on either Picture, the Projection of Objects is determined in the very fame Manner. But if there fhould appear any Difficulty, it cannot be in the Operation, but in confidering what Objects are proper and what not for fuch kind of Pictures; and the Situation of thofe Objects. For inftance, to reprefent a Landfkip or any Objects which are fuppofed to be upon the Ground, is extremely improper for a Ceiling; for fince the Picture is always fuppofed parallel to the Ground, and the Eye is placed between the original Object and the Picture, therefore the Rays of Light in their Paffage from original Objects to the Eye, will not be cut by the Picture, and confequently fuch Objects can have no Projections upon the Picture; for which Reafon they ought not to be reprefented. But any Objects which may reafonably be fuppofed to exift in the Air, or any Story which can be fupported either by Hiftory or Fable, may be reprefented with the greateft Propriety; as may likewife feveral Parts of Architecture, which may ferve either for Ornament, or be ufeful as to the main Defign. And in regard to the Situation of Objects, they are generally fuppofed to be erect, and therefore I fhall principally confider them in that Situation; which will be fufficient for our Purpofe, as it will give the Reader a very clear Idea of all that can be faid upon the Subject; and which, together with the Examples under this Head in the Second

Book, will enable him to find the Reprefentation of all Objects upon a Ceiling with the fame Facility as he can determine thofe upon an upright Picture.
Fig. 73.
Let KMNO be the Ground, E the Eye, EH its Height, DGLP the Picture, (which we will fuppofe a Ceiling) C the Center of the Picture, CE its Diftance, and GACL a Plane perpendicular to the Picture whofe Reprefentation is required.

From the Section GL, draw GC, LC; then from A and C draw $\mathrm{AE}, \mathrm{CE}$, cutting $\mathrm{GC}, \mathrm{LC}$, in a and c ; then draw a c , which will compleat Gac , the Reprefentation of the original GACL; which will, to an Eye placed in E, appear to be erect.

Let us now turn the Figure in fuch a Manner that the Picture may become an upright one; then ACMK is the Ground Plane, E the Eye, EC its Difance, EI its Height, $C$ the Center of the Picture, VL the horizontal Line, and GacL the Reprefentation of GACL, which lies upon the Ground. From hence then it is evident, that in order to determine the Appearance of any perpendicular Plane upon the parallel Picture, we muft proceed in the very fame Manner as in finding the Reprefentation of an Object which lies flat upon the Ground in the perpendicular Picture; for in both Cafes, the original Plane ACLG is perpendicular to the Picture, only the Situation of the Picture is different in regard to the Eye, and therefore the Reprefentation in both Cafes will be the fame, as is manifeft by infpecting the Figure.
But fuppofe the original Plane be parallel to the Picture; then the Reprefentation will be like the Original, and muft be found by the fame Rules as Objects thus fituated are determined upon the perpendicular Picture.
Fig. 75.
Thus, let LGPO be the Picture, E the Eye, C the Center, and $C E$ its Diftance, and $A B C D$ the original Plane parallel to the Picture.

From A, B, C, D, draw Lines perpendicular to the Picture, interfecting it in the Points $G, L, P, O$; then from thofe Points draw Lines to the Center of the Picture; and from the Points A, B, C, D, draw Lines to E, which will interfect GC, LC, PC, and $O C$, in $a, b, c, d$, and thereby determine the Reprefentation required.

Now let us turn this Figure alfo, and call ABMK the Ground Plane; then this Picture is an upright one, and the Reprefentation $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$, of the parallel Plane ABCD, in either Situation of the Picture is the fame; and confequently the Reprefentation of all paralle!

## Of the Parallel Picture.

parallel Objects are to be determined after the fame Manner as in the upright Picture.

Now, fince the Rules for drawing the Appearance of Objects upon the parallel Picture, are exactly the fame as thofe for drawing the Appearance of Objects upon the perpendicular Picture, it follows, that the fame Rules will do in both Cafes, and therefore the Artift has nothing more to remember than this, viz. thofe Objects which in the parallel Picture are to be reprefented as erect, muft be determined as thofe which lie flat upon the Ground in the perpendicular Picture; thofe which are parallel in one Picture, as thofe which are parallel in the other; and thofe which are oblique, after the fame Manner: Or in other Words, however original Planes are fituated, the Reprefentations of them muft always be determined by imagining a Plane to pafs through the Eye parallel to thofe Planes, which will give their feveral vanifhing Lines, from which the whole Reprefentation may be compleated. Thus, the Plane FGVL, which paffes through the Eye E, parallel to the Fig. 73: original Plane ACLG, produces the vanifhing Line VL of that Plane; and therefore having the Diftance EC of that vanifhing Line, the Reprefentation of any Lines which can be drawn in the original Plane are eafily found alfo.

And here we may obferve, that if the original Plane ACLG were infinitely extended, the Triangle GLC would be its indefinite Reprefentation, and confequently the Appearance of all Lines which can be drawn in that original Plane, will be fomewhere within that Triangle. And fo likewife, if perpendicular Planes are erected on the other Sides LP, PD, DG, of the Picture, their indefinite Reprefentations will be the feveral Triangles LCP, PCD, and DCG, and the Center C will be their common vanifhing Point.--For draw the original Plane ACLG upon the Side LG of the Fig. 74* Picture, and let every thing elfe remain as in the former Figure. ---Through E draw the Plane FHGLV, parallel to the Plane ACLG, which will cut the Picture in VL; then is VL the vanifhing Line of that Plane. Again, from E draw EC, perpendicular to VL; then is C the Center of the Picture. And fince EC is paraliel to AG, BS and CL, therefore C is the vanifhing Point of thofe Lines; and therefore, from C , the Center of the Picture, draw Lines to G, S, L; and from A, B, C, draw Lines to E , which will cut the former Lines in the Points $\mathrm{a}, \mathrm{b}, \mathrm{c}$; then is a G the Reprefentation of A G, bS of BS, and cL of CL; and GacL is the whole Reprefentation of the original Plane ACLG.

And after the fame Manner any other Lines, as $x z$, may be found upon the Picture.

And from hence alfo, we may obferve, that if perpendicular Planes are fet on each Side of the Picture, the Reprefentation of thofe Planes will appear like the Sides of a Room continued upwards; from whence it follows, that by fuch Deceptions as this, a Room may be made to appear of any Height, by drawing a Reprefentation of this Kind upon a Ceiling with Accuracy and Judgment, and viewing it from the proper Point. One Example of which I fhall give in this Place, by way of Practice, and then refer the Reader again to the fecond Book for more Examples of this Sort.
Fig. 76. Let GLPO be a Ceiling, E the Eye, EC its Diftance, and C the Center of the Picture.

Through the Center C draw Lines parallel to LP, LG, and continue them at pleafure; then with the Diftance CE defcribe a Circle, cutting thofe Lines in D, F, H: Then DCH is the vanifhing Line for the original Planes, which ftand upon the Sides GL and OP; and ECF is the vanifhing Line of the Planes which ftand upon the Sides GO and LP; and the feveral Lines EC, DC, FC and HC, are the Ditance of the Eye from thofe Lines. Having fettled the vanifhing Lines of the four Sides, their Center and Diftance, it matters not upon which Side we begin to work; for upon any Side, as G L, draw out one of the original Planes, as ACLG, and upon it draw the Lines $\mathrm{XZ}, \mathrm{BS}$, which will make it like the Plane ACLG, Fig. 74. From the feveral Sections $G, S, L$, draw Lines to $C$; and from $A, B, C$, draw Lines to $E$, cutting GC, SC, LC, in the Points $a, b, c$; then from a to $c$ draw ac , and then will G a be the Reprefentation of $\mathrm{GA}, \mathrm{Sb}$ of SB, and Lc of LC: Therefore, GacL is the Reprefentation of the whole original Plane GACL, and the Triangle GCL is the Reprefentation of that Plane infinitely extended.---In like Manner xz is the Reprefentation of its Original X Z.

Or the Operation may be fhortned thus. From the extreme Point B of any Perpendicular in the original Plane, draw a Line, BI, at pleafure, cutting the Section in I; then from E draw EK parallel thereto, cutting the vanifhing Line DH in K ; from the Section S, of the Perpendicular SB, draw SC; and from the Section I draw IK, cutting SB in b: Then is bS the Depth of the Reprefentation; therefore, by drawing GC, LC, and by drawing a Line through $b$, parallel to $G L$, the Thing propofed is done.

Now, in order to transfer this Reprefentation unto all the other Sides, proceed thus.

From O and P draw Lines to the Center C; then will the remaining Part of the Ceiling be divided into three Triangles, GCO, $\mathrm{OCP}_{5}, \mathrm{PCL}$; which Triangles may reprefent three Planes perpendicular to the Ceiling, infinitely extended, and at rightingles with each other; and GC, OC, PC, and LC, reprefent the joining of thofe Planes: For GC and LC are the Reprefentations of GA and LC infinitely extended; and therefore, having found the Depth (as Ga) of the Reprefentation of any given Plane, as above, from the Point a, which determines that Depth, draw a Line, as a $e_{2}$ parallel to OG ; and from e, where a e cuts OC , draw another Line ed parallel to OP; and from d, where ed cuts PC, draw d c, which will cut $L C$ in $c$; then will $G$ aeO, OedP, and PdcL, be the Reprefentations of three perpendicular Planes of the fame Height as ACLG, and fituated in the fame Manner; that is, upon the feveral Sides GO, OP, and PL; and confequently, to an Eye placed at E, and at the Diftance EC, the Sides of a Room will appear to be continued above the Ceiling by the Length of the Perpendicular GA, i.e. the Height of the original Plane ACLG.

## S E C T. II.

## Of the INCIINED DICTURE.

IHave before obferved, that by an inclined Picture, I would be underftood to mean when the Perfpective Plane is neither perpendicular nor parallel to the Ground, but inclined to it. Indeed, this Situation of the Picture is very feldom made ufe of, yet as there are fome Cafes which may require the Knowledge of this kind of Perfpective, I have therefore given it a Place in this Work,

Let OPH be the Ground or original Plane, HLGL the Pic- Fig $77^{\circ}$ ture, inclin'd to the Ground Plane at the Angle PLL ; and let E be the Eye, EH its Height, and H its Seat upon the Ground.

Continue the Picture HLGL downwards at pleafure, as GLFO. From the Seat H of the Eye draw HS perpendicular to the Section GL, cutting GL in $S$; then through $S$ draw $S D$, perpendicular to GL alfo, and continue it at pleafure towards FO; and then from E draw ED, parallel to HS, cutting the Picture in D, and continue EH 'till it cuts DS in V; then from V draw VI, parallel to ED, and from D draw D I, parallel to EV: And then will EDIV be a Plane

Plane which paffes through the Eye perpendicular to the Ground Plane OPH, interfecting the Picture in the Line DV; and therefore the Section DV will be the vanifhing Line of all Planes that are perpendicular to the Ground Plane and parallel to the Plane EDIV; and for the fame Reafon, $V$ will be the vanifhing Point of all Lines that are perpendicular to the Ground Plane OPH, becaufe EV which is drawn through the Eye parallel to thofe Lines, will cut the Picture in the Point V: For as in the upright, or parallel Picture, fo alfo in this, the vanifhing Line of any original Plane muft be determined, by imagining a Plane to pafs thro the Eye parallel to that original Plane 'till it cuts the Picture. And fo alfo in regard to the Center and Diftance of the Picture, or the Center and Diftance of a vanifhing Line; the firft is found by drawing a Line from the Eye, as EC, perpendicular to the Picture, and the latter, by drawing a Line from the Eye, as ED, perpendicular to that vanifhing Line: The Method for doing either is as follows.

## 1. For the Center and Difance of the Picture.

Having continued the Picture downwards as above directed, and drawn the vertical Plane EDIV; from E, draw EC, perpendicular to the Section DV; then will C be the Center of the Picture, and CE its Diftance: For fince the vertical Plane cuts the Picture at right Angles, and fince EC is in that Plane, and perpendicular to the Section DV, therefore EC is perpendicular to the Picture alfo, and confequently C is the Center of the Picture, and CE its Diftance.
2. For the Center and Diftance of a vanibing Line.

Let the Plane ABHL pafs through the Eye E, parallel to the Ground Plane OPH, and it will cut the Picture in HL, which Line HL is the vanifhing Line of the original Plane OPH; and if from $E$, a Line, as $E D$, be drawn perpendicular to $H L$, then $D$, where it cuts HL, is the Center of that vanifhing Line, and DE is its Diftance.
Fig. 78. Now, let it be required to find the Reprefentation of the original Plane ABGL upon the inclined Picture GLHL; and let E be the Eye, H its Seat upon the original Plane, EC its Diftance, and C the Center of the Picture.

From H, the Seat of the Eye, draw HS, perpendicular to the Section GL; from S, draw SD perpendicular to GL, and continue it at pleafure ; then from the Eye E, draw. ED parallel to HS, cutting SD in D; finally, through D, draw HL, parallel to

## Of the Inclined Picture.

$G L$, then is HL the vanihing Line of the original Plane ABGL, and $D$ is the vanifhing Point of the Sides $A G, B L$; therefore, from $G$ and $L$ draw $G D, L D$, and from $A$ and $B$ draw $A E, B E$, cutting GD, LD, in the Points $a$ and $b$; then is GabL the Reprefentation of the original Plane GABL.

To apply this to Practice.--Let GLNM be the Picture laid Fig. 80, flat, as in fome of the preceding Figures.---Bifect the Bottom GL; and draw CD perpendicular thereto, and continue it at pleafure: Then from the 7.8 th Figure take SC, CD, and transfer them unto cD in this Figure, beginning at the Point $c$; draw HL; then is C the Center of the Picture, cD the Height of the vanifhing Line, and $D$ its Center. Again, make $D \mathbb{E}$ equal to the Diftance of the Eye, and AG equal to the Length of the original Plane, (that is, equal to AG Fig. $7^{78}$.) then from $G$ and $L$ draw GD, LD, and from A draw A Le, cutting $G D$ in $a$; finally, from a draw $a b$, parallel to GL; which will compleat a Reprefentation GabL。 exactly like G-a bL Fig, 78 .

Or it may be done thus.--From the Center $C$ draw $C E$, parallel to $G L$; and make $C E$ equal to the Diftance of the Picture, and ED equal to the Diftance of the vanifhing Line HL; then from D, with, the Radius DE, defcribe the Arc ELEH; and from G, with the Radius GA, defrribe the Arc Ac; and then from c and H draw Hc, which will cut GD in a, and give the Depth of the Reprefentation ; from whence the whole may be compleated.

In like Manner, let it be demanded to find the Projection of a Fig. 7g. Line $A B$, which ftands perpendicular to the Ground Plane OPH.

From B, the Seat of the Line AB, draw a Line BH to the Scat of the Eye H ; and from V draw Vd , through the Section c , and continue it at pleafure; then from $A$ and $B$ draw Lines to the Eye $E$, cutting $V d$ in $a$ and $b$; and then is $a b$ the Reprefentation or the Original $A B$. For fince $E V$ is parallel to the Original $A B$, Point of AB , and of all other Lines which are parallel to AB . "And if we imagine a Plane ABHE to pafs through the original $A B$, and the Line HE, it will cut the Picture in $c a ;$ and therefore, fince the Rays AE and BE are in that Plane, the Section $a b$ will be the Reprefentation of AB.

To apply this to Practice.---Let MNGL be the Picture, laid Fig. 81 . flat as before. Then C is its Center, CE its Diftance, V the vanifhing Point of Lines perpendicular to the Ground Plane, HL the vanifhing Line of Planes parallel to the Ground Plane, $D$ the

Center of that vanifhing Line, and DE its Diftance. Now, let it be required to find the Reprefentation of a fquare Plane which ftands perpendicular to the Ground Plarie, having one Side, $a b_{2}$ of the Reprefentation given.

From $D$, the Center of the vanifhing Line $H L$, and with theDiftance DE, defcribe an Arc ELFH, cutting the vanifhing Line in H and L ; then is H the vanifhing Point of the Sides ad, bc : Therefore, draw aH, bH, and from H draw HV ; fo will HV be the vanifhing Line of a Plane perpendicular to the Ground; and by finding A (the vanifhing Point of the Diagonal of a Square) the whole Reprefentation may be determined.

The 82d Figure reprefents a Cube upon the inclined Picture : For having determined the Appearance of one Face abcd, as in the lait Figure, the whole Reprefentation may be compleated, by means of the vanifhing Lines HL, HV, and LV, and the vanifh ing Points of the Diagonals, B, D,G.

1 have hitherto confidered the Picture as reclined from the Eye; let us now fuppofe it to be inclined to the Eye, as in Fig. 83. where E is the Eye, LV the Picture, C its Center, CE its Diftance, $V$ the vanifhing Point of Lines perpendicular to the Ground, and DL the vanifhing Line of Planes parallel to the Ground.-In the 84 th Figure the Picture is laid flat, and the Reprefentation of one Face of a Cube is determined: "And in the 85 th Figure, the Projection of the whole Cube is compleated.---Thefe Figures need no Explanation, "being only as it were the Reverfe of the others; and therefore a little Attention muft render them extremely obvious.

From hence then it follows, "that the Method of determining the Reprefentation of a Cube upon an inclined Picture, is exactly the fame as in finding the Appearance of a Cube any ways inclined to the Ground; and therefore the Rules which ferve for the one will ferve for the other alfo: For which Reafon the Learnes is defired to compare this with what has been faid in Seet. 53 Chap. 3.




## Of Vavited Roore Domes Era

S E C T. III, Of VAUETED ROOFS, DOMES Br,

TO draw Perfpective Reprefentations upon vaulted Roofs, Domes, or any other uneven Surfaces, has always been efteemed a Work of great Difficulty; and among all the Methods which have been given us for this Purpofe by different Authors, none feems fo practicable as that by Mr. Hamilton, in his ingenious Treatife intitled Stereography; where he directs us to *Reticulate the propofed Surface, in fuch a Manner as may be beft fuited to its Shape, and can with the moft Eafe be done; then to draw out, on a Plane properly chofen, a Picture of the intended Defign, by way of Model; after which, to draw on this Model, the Image of that Reticulation, by the common Rules of Perfpective; which will divide the Defign on the Model, into fuch Parts, as are proper to be transferred into each correfponding Cell of the original Reticulation; and finally, by means of this Reticulation, to transfer the Work unto the Dome or Roof, in the fame Manner as one Picture is copied from another, by the common Methods of Reticulation.

Thus, fuppofe it was required to paint fome perfective Repreentation upon a vaulted Roof, HOIKPG.

Let this Figure be a Model drawn out upon Paper, of a vaulted Fig. $8 G_{\text {g }}$ Roof; and let GHIK reprefent a Plane, which is fuppoied to pafs through the Foot of the Arch, parallel to the Horizon.

Now, if we fuppofe the Spectator's Eye to be placed directly under the Middle of it at E , and then imagine a Plane $\mathrm{A} B \mathrm{LQM}$ to pafs through the Eye, perpendicular to the Ground Plane $A B$, it will cut the Picture in ML; and therefore, by drawing EC perpendicular to the Section ML, we fhall have C for the Center of a parallel Picture, and CE for its Diftance.----Let us next divide the Roof into any Number of Squares, or Parallelograms, as in the Figure ; and then imagine a Line to be drawn from the Angle of every Square to the Eye E; and it muft appear extremely evident, that the Sections of thefe Lines with the Plane, or parallel Picture GHIK, will be the Projection of thofe Points upon the Picture ; and it muft alfo appear as obvious, that, when the Projection of the Angle of every Square is determined upon the Pic-

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## Of VAOLTED ROOFA, DOMES, EC.

ture, the whole Reprefentation of thofe Squares may eafily be compleated. But farther, fince the petpendicular Plane ABLQM paffes through the Eye, and cuts the Picture in a frait Line; therefore the Projection MCL, of the Arch MQL, will be a ftrait Line upon the Picture; but the Projection of all the other Arches, $i R_{4}$, HOI, $\mathcal{B}^{\circ}$. will be curve Lines. Again, fince the tranfverfe itrait Lines $75, \mathrm{PO}, 68$, are parallel amongft themfelves, and are alfo parallel to the Picture ; therefore the Reprefentation of thofe Lines upon the Picture, will be ftrait Lines, and parallel to each other.
Thefe Things being premifed, let ns now fuppofe this Figure removed to the 87 th Figure.---About the Arch HOI, defcribe the Parallelogram HIPN; and through the Points 5, 0,6 , draw the Lines $17,08,29$, perpendicular to the Picture, and cutting the Picture in the Points 7, 8,9; then through 5, 6, draw QR parallel to HI, and from the feveral Sections H, $7,8,9, \mathrm{I}$, draw Lines to C , and from $\mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}$, draw Lines to the Eye E , which will determine the Projection of the Parallelogram; by which means the Reprefentation HaocI , of the Arch HOI, may be compleated: After the fame Manner, the Projection of all the other Arches may be found; but as one is fufficient for our Purpofe, we will now fuppofe this parallel Picture to be laid down flat in the 89 th Figure, where C is the Center, CE the Difance of the Picture, and $\mathrm{H}, 7,8,9, \mathrm{I}$, the Sections of the Perpendiculars $\mathrm{NH}, \mathrm{I}, 7, \mathcal{E}_{\mathrm{c}}$. in Fig. 87.

Continue IH (Fig. 89,) at pleafure, towards N , and make HQN in this Figure equal to HQN in the 87 th Figure; then from $\mathrm{H}, 7,8,9, \mathrm{I}$, draw Lines to C , and from N and O draw Lines to E , which will cut HC in a and t , and thereby give the Depth of the Parallelogram HnpI; by which Means the Points H, a, o, c, I, will be determined : Which being fo many Points in the Reprefentation of the Curve, they will be a fufficient Guide for drawing it, as in the Figure. After the fame Manner, the Reprefentation of the other Front Arch is to be found: From whence it follows, that the Projection of the whole curved Roof upon this parallel Picture, will be contained within the two curved Lines HoI, GgK, and the two ftrait Lines GH and IK; and therefore GHoIKg is the whole Space allotted for the Defign. Now having determined this Space, let us next find the Projection of the feveral Squares which were fuppofed to be drawn upon the original Roof.

From

## Of Vaulted Roofs, Dooms, ©ch

From a, o, c, draw Lines parallel to the Side GH, or IK ; then will af,og, cd , be the Projections of the tranfverfe Divifions (or ftrait Lirtes) which are parallel to the Picture; and by dividing the feveral Lines HG, af, $\mathrm{og}, \mathrm{cd}$, and IK, into four equal Paits, we fhall have the Points given, through which the other Curve Lines are to be drawn, as in Fig. 88; by which Means the whole Reprefentation may be compleated.

If it be required to paint any Perfpective Reprefentation npon a Dome, that alfo may be done after the fame Manner, viz. by imagining the Dome to be divided into feveral perpendicular Sections, drawn at equal Diftances from the Bafe, through the Center of the Dome; and by fuppofing thofe Sections to be cut by other Sections, which are made by Planes that are fuppofed to pafs through the Dome parallel to the Horizon: Then by making a Model upon Paper in a given Proportion, and taking the Diftance of the Eye accordingly, we may find the Projection of thofe Sections upon the parallel Plane, as in the former Figures: For then we fhall have a parallel Picture, which we fuppofe paffes under the Bottom of the Dome, properly reticulated, and by that Means, whatever is drawn upon it, may be transferr'd unto the real Dome or Cupola.

Thus, Iet ABDE, Fig. 90, reprefent the circular Plane (or parallel Picture) which we fuppofe to lie under the Bottom of the Dome; and let AacegfdbB, Fig. 91, reprefent one of the perpendicular Sections above-mention'd; and let us imagine the Dome to be divided perpendicularly by four of thefe Planes, and horizontally by four Planes, the Sections of which horizontal Planes are exprefled by $A B, a b, c d$, ef: Then let us divide the Circumference of the Plane, or Picture, ABDE (Fig. 90) into eight equal Parts, and from each Part draw Lines through the Center C ; and then will thefe ftrait Lines be the Projections of the perpendicular Sections upon the Picture. And in order to find the Projections of the parallel Sections; from C, the Center of the Picture, draw CE Fig. 9r. perpendicular to $A B$, and equal to the Diftance of the Eye; then from $a b, c d$, ef, and $g$, draw Lines to the Eye $E$, which cutting the Picture, will give 16 for the Projection of $a b, 25$ for that of $c d, 34$ for that of ef, and $C$ the Center of the Picture for g the Center of the Dome; therefore, from the Line AB transfer the feveral Divifions $A_{1}, 12, \mathcal{E} c$. unto the Line $A B$ in the 90 th Figure ; and from the Point C, defcribe the feveral concentric Circles through the Points $1,2,3$; and fo will the whole Picture be properly divided for the Work: For each Reticulation upon the

Picture, is the exact Projection of its correfponding and original Reticulation upon the Dome; and therefore, all that now remains is, only to divide the Picture into an agreeable Number of Parts, and to confider each Part as a parallel Picture, whofe perpendicular Sides will vanifh into the Center of the Picture; and to be always careful to take the Center of the Model perpendicular to the fuppofed Place of the Eye; and the Diftance to be work'd with, muft be the fame as that between the Eye and the Plane AB, Fig. 9r, as well for defcribing the Model itfelf, as for the Reticulation.

We have hitherto confidered the Eye as placed under the Center of the Dome, in which Cafe the Reticulation upon the parallel Picture is done with great Eafe: But if it were placed obliquely, the Reticulation would become a little more troublefome; in regard that in fuch a Pofition of the Eye, the perpendicular Sections of the Dome would not form ftrait Lines upon the parallel Plane, but Curves.

Thus, let AgB , be a perpendicular Section of the Dome, and ab, cd, ef, its Sections with the horizontal Circles, as before; and let $E$ be the Place of the Eye.

Then Lines drawn from $E$, to the Vertex $g$ of the Dome, and to the Centers and either Extremity b, d, $f$, of the horizontal Diameters, will cut the Bafe of the Dome $A B$ in correfponding Points; which being transferred by Perpendiculars to the Diameter AB of the parallel Plane, will give the apparent Vortex C, and the Centers and Radii of the Images of the horizontal Circles, on the parallel Plane ; and thefe being drawn, and each divided into the fame Number of equal Parts, as the Bafe of the Dome is fuppofed :to be, Curve Lines drawn through the correfponding Divifions of thefe Circles, will give the Projections of the feveral perpendicular Sections of the Dome, as in the Figure.

For as the horizontal Circles are all fuppofed parallel to the circular Plane ADBE, it is evident their Projections will ftill remain Circles, and their Subdivifions will be equal, like thofe of their Originals.

And here all the perpendicular Sections of the Dome form Curves upon the parallel Plane, except the Section AgB, which is projected into the ftrait Line AB, the Eye being fuppofed to lye in the Plane of that Section. But in Painting on curvilinear Grounds, the moft direft Situation of the Eye ought always to be cholen, that the Defign, when painted, may appear the more agreeably; and indced, in all fuch Works, the Defign ought, as:much as pof-

XVII .

Of VAULTED ROOFS, DOMES, EC.
fible, to be fuited to the Shape of the Surfaces, and to confift principally of ornamental Architecture fitted to it, (putting the Hiftorical Part into fmall Compartments, to be difpofed in proper Places) or elfe of fome Aerial View, where the Sky and Clouds $x_{x}$ with other Objects proper for that Situation, may be defcribed; in which Cafe, the principal Objects not being confined to regular: Figures, there will be lefs Danger of their appearing diftorted by the Shape of the Surface painted upon.

But when a Cupola, Dome, or Vault, is to be defcribed on 2 . flat Ground, there may be a greater Liberty taken in placing the Eye; which may have either a direct or oblique Pofition, as the Artift judges beft for the View he intends to reprefent, and will not be liable to thofe Inconveniencies which attend Painting upon an uneven Ground.

## C H A P. V.

## The Perspective of Shadows.

THE Meaning of the Word Shadow is too obvious to need any Explanation; and therefore I fhall not trouble the Reader about its Etymology, nor Thall I confider that infinite Variety of Shadows which may be projected by different Planes; but proceed to Shew, that the Perfpective of Shadows upon the Picture, is to be determined after the fame Manner as the Perfpective of Objects, being founded upon the fame Principles, and deducible from the fame Rules: It is therefore very furprizing, that almoft every Author who has handled this Part of Perfpective, fhould have committed fuch egregious Miftakes, in giving füch Rules as are falfe in Theory, and in Practice the moft abfurd.

But to proceed. All Shadows are produced by the Interpofition of fome opake Objects, which ftop the Progrefs of the Rays of Light in their direct Courfe from any luminous Body or Point. And fince the Rays of Light do always proceed in ftrait Lines, therefore, when they pafs over the Extremities of an Object, they leave a Space unilluminated, which Space is called, the Projection ef the Sbadow of that Object: And 'tis the Bufiness of this Part of Perfpective, to determine the Appearance of that Projection upon the Picture. In order to do which, we muft firft confider whether the Light be fuppofed to come from the Sun, a Candle, or any other luminous Point: If from the Sun, then, from its immenfe Diftance with refpect to us, the Rays may be confidered as parallel; but if from a Candle, as flowing from a Point in a diverging Manner.

And in regard to the Theory of the Perfpective of Shadows; there needs but little more to be faid than what has been already advanced upon the Perfpective of Objects: For fince every Ray of Light is to be confidered as a ftrait Line, that Line may be conceived to lie in fome Plane; and therefore, if the Reprefentation of that Plane can be eafily found upon the Picture, the Reprefentation of a Line which is in that Plane, may be eafily found alfo.

And here let us obferve, that the Planes in which I thall fuppofe the Rays of Light to be, will always be confider'd as perpendicular
Of SHADOWS by the SUN.
to the. Horizon, as that will be fitter for our Purpofe, and render the Thing more intelligible.

If the Rays of Light come from the Sun in Planes parallel to the Picture, they then can have no vanifhing Point; in this Cafe, therefore, the Shadows will be parallel in the Picture: But if they come in Planes not paralleil to the Picture, then becaufe a Line drawn from the Eye parallel to thofe Rays will cut the Picture in fome one Point, therefore they will have a vanifhing Point upon the Picture, which will be the common vanifhing Point for all the Rays in that Direction, whether they be all in one Plane or in any Number of Planes, provided thofe Planes are parallel to one another.
S E CT. I.
Of SHADOWS projected by the SUN.

IET HP reprefent a Plane parallel to the Horizon, or, if Fig. 93. L you pleafe, call it the Ground Plane, and let $A B C D$ reprefent a Plane of parallel Rays, as EL, Dd, © c. each Ray making an Angle, RLA, with the Ground Plane.

Now, in order to find the Perfpective of any Shadow upon the Picture, two Things are neceffary to be given;* viz. the Inclination, or Angle, which any Syftem of Rays makes with the Ground Plane, and the Situation of the Plane (in refpect to the Picture) in which thofe Rays are fuppofed to be. As to the Angle of Inclination, that may be given by a fingle Ray only; for fince the Rays RL, Dd, $\mathcal{E}_{c}$. are all fuppofed parallel amongft themfelves, therefore the Angle RLA, which any fingle Ray RL makes with the Plane HP, is common to all the reft: And as to the Situation of the Plane of Rays, that is to be chofen at the Difcretion of the Artift, fo as to be moft productive of Effect as to his main Defign.

## Lemmai.

If the Rays of Light come from behind the Picture towards the Spectator's Eye, then the vanifhing Point of thofe Rays will be above the horizontal Line.

Let PQ be the Ground Plane, GO the Picture, E the Eye, EC Fig. 95: the Diftance of the Picture, C the Center of the Picture, HC the

[^15]
## Of SHADOWS by the 9 UN

horizontal Line, ABOD a Plane of parallel Rays interfecting the picture at right Angles in the Line BO; and let RL be a Ray of Light, and R LA the Angle of Inclination which the Rays make with the Ground Plane.

Through the Eye E, draw EF parallel to any of the Rays, as RL, cutting the Picture in $F$; then is $F$ the vanifhing Point of all the Rays of Light; for EF being parallel to one Ray, is parallel to all the reft.--Now, fince the Plane of Rays ABOD is perpendicular to the Picture, and paffes through C the Center of the Picture, therefore the Plane BFEH, which paffes through the Eye, will be perpendicular to the Picture, and will pafs thro its Center alfo; and therefore BO, the common Section of thefe two Planes, will be the indefinite Reprefentation of the Plane ABOD; and confequently, F , where EF cuts the Picture, will be the vanifhing Point of the Rays RL, $\mathcal{E} c$.
COROL. I.

Since the Plane of Rays paffes through the Center of the Picture, the vanifhing Point of the Rays will be in a Line drawn from the Center of the Picture perpendicular to the horizontal Line.

$$
\text { COROL. } 2 \text {. }
$$

From hence alfo we may perceive, that C may be the Reprefentation of the Seat of the luminous Point; for the Seat of the real Luminary is fuppofed to be in a Plane parallel to the Plane of the Horizon; and therefore, if we confider $A$, the Seat of R, as at an immenfe Diftance, and fuppofe $R$ a real luminous Point, then will C be the Reprefentation of the Seat R ; that is, the Reprefentation of the Seat of a luminous Point upon the Picture, which is fuppofed to be at an immenfe Diftance from it: Or, in other Words, fince C is the vanifhing Point of LA infinitely extended, therefore, it is alfo the vanifhing Point of any Point in that Line at an immenfe Diftance.

$$
\text { COROI. } 3 \text {. }
$$

And here likewife we may obferve, that in order to find the vanifhing Point of a Ray of Light, or of any Number of parallel Rays, we need only have the Angle of Inclination given; then by fetting off the Diftance of the Picture upon the horizontal Line, and making an Angle at that Point of Diftance with the horizontal Line, equal to the given Angle of Inclination, we may determine
Of SHADOWS by the SUN:
the vanifhing Point of thofe Rays; as is fhewn in the 98 th Figure ; which will be more fully explained hereafter.

$$
\text { LEMMA } 2 \text {. }
$$

In the laft Figure we confider'd the Rays as coming in a Plane perpendicular to the Picture; we will now fuppofe them to come in a Plane oblique with the Picture.

Let ABOD be a Plane of Rays which cuts the Picture obliquely Fig. 96i in the Line OB , every thing elfe remaining as in the former Figure.

Through the Eye E draw the Plane HLLE parallel to the Plane of Rays, cutting the Picture in LL; then continue LL upwards beyond $F$, at pleafure, and from E draw EF parallel to the Ray of Light RL; then is F the vanifhing Point of that Ray, $\mathcal{E}_{6} c$.
Corol. I.

From hence it follows, that when the Light comes from behind the Picture, the Shadows of Objects will be thrown towards the Bottom of the Picture.

$$
\text { LEMMA } 3
$$

When the Rays come from behind the Spectator's Eye towards the Picture, (that is, when the Spectator is between the real Luminary and the Picture) then the vanifhing Point of thofe Rays will be below the horizontal Line.

Let FHIL be the Picture, E the Eye, C the Center of the Pic- Fig. 97. ture, and EC its Diftance; and let ABOD be a Plane of parallel Rays whofe Seat upon the Ground Plane is in the right Line LH continued: Or in other Words, fuppofe the Plane of Rays was continued towards the Picture in the Line BL, it would pafs thro' the Eye E, and would cut the Picture in the Line LI.

Through the Eye E, and its Seat H, draw EC, HL, parallel to $A B$ or $C D$; and from $L$, where HL cuts, the Section GL, draw LC parallel to AD; then is EHLC a perpendicular Plane which paffes through the Eye parallel to the Plane of Rays ABOD, cutting the Picture in LC; therefore LC continued will be the indefinite Reprefentation of the Plane ABOD, and it will alfo be the vanifhing Line of all the Rays which can come in that Plane; and if EF be drawn parallel to R $L_{2}$, then is $F$ the vanifhing Point of that Ray, and C the Reprefentation of its Seat upon the hori zontal Line HL。

For fuppofe the Plane ABOD to be tranfpofed into the Line: XZ , then it will be like the Plane ABOD in the $95^{\text {th }}$ Figure, with this Difference only, that the Rays coming in a contrary Direction, will have their vanifhing Point upon the Picture below the horizontal Line.

## Coroleary.

When the Light comes from behind the Spectator's Eye towards the Picture, the Shadows of Objects upon the Picture will be thrown towards the horizontal Line ; and fince the Light is generally fuppofed to come upon the Front of the Picture, and not from behind it, therefore thefe Kind of Shadows are moft generally ufed:

I fhould have been more particular in the Explanation of this Figure, if there appeared the leaft Difficulty to me in underftanding it: Indeed, as the Eye is fuppofed to be between the Picture and the original Object, it may feem to contradict our gencral Definition of Perfpective, in which we have always confider'd the Picture as placed between the Eye and the original Object; and therefore this Lemma may appear not to be fo aptly drawn from the preceding Theorems* as it really ought to be: Yet, fince the Method for determining a vanifhing Line, or Point, is the fame in either Cafe, viz. by imagining a Plane to pafs through the Eye, parallel to the original Plane, 'till it cuts the Picture, $\mathcal{O}^{\circ}$ c. I have, therefore, only explained that fingle Article, and endeavoured to make myfelf underftood, in the moft familiar Manner; not much regarding ffrict mathematical Demonftration, nor yet that Order or Method which would be neceffary were this Treatife purely Mathematical.

## LEMMA 4.

When the Rays of Light come in Planes parallel to the Picture, they can have no vanifhing Point; becaufe a Plane which paffes through the Eye parallel to thofe Planes, and which in other Cafes would cut the Picture, and thereby produce a vanifhing Line, in this Cafe can never cut the Picture, and therefore cannot produce any vanifhing Line: From whence it follows, that when the Rays come in this Direction, the Appearance of their Shadows upon the Picture will be parallel, for the very fame Reafon that the Repre-

> Of SHADOWS by the SUN.
fentation of any original Plane which is parallel to the Picture, is exactly like its Original.

We will now give fome general Rules for applying to Practice what has been faid upon this Head. In order to do which, let AB reprefent a Picture laid flat, as in the preceding Examples; Fig. 9 s. and let HL be the horizontal Line, $\mathbf{C}$ the Center of the Picture, and $C E$ its Diftance.
METHOD I.

To find the vanifhing Point of a Ray of Light, when it is fuppofed to come from behind the Picture towards the Spectator's Eye, in a Plane like ABCD, Fig. 95, which cuts the Picture in its Center;

Any where apart, draw NP parallel to the horizontal Line HL, Fig. 98. and draw NO, at pleafure, for the Ray of Light; then is ONP the Aingle of Inclination.---Through C the Center of the Picture, draw EK perpendicular to HL, and continue it at pleafure ; then make $C H$ equal to the Diftance EC, and from $H$ draw HD parallel to the Ray NO, cutting CE in D; and then is D the vanifhing Point of the Rays of Light. For fince EK is the vanifhing Line of the Plane of Rays, C the Center of that vanifhing Line, and CH equal to its Diftance, therefore $H$ may be confidered as the Eye; and confequently, fince HD is drawn from that Point parallel to the original Line NO, the Point $D$, where it cuts the vanifhing Line ED, is the vanifhing Point of that original Line.

$$
\text { METHOD } 2 .
$$

When the Rays come from before the Picture, as in Fig. 97 ;
Every Thing remaining as before, --Let TW be a Ray of Light, Fig. 98. and VTW its Angle of Inclination.---From H draw HK, parallel to the Ray. TW, cutting the vanifhing Line EK in K ; then is $K$ the vanifhing Point required.

$$
\text { METHOD } 3 \text {. }
$$

When a Ray of Light comes from behind the Picture in a Plane oblique with the Picture, as in Fig. 96;

Let IG be the vanifhing Line of a Plane of Rays, RS a Ray Fig. 98. of Light, and RSQ its Angle of Inclination.---Continue the horizontal Line beyond $L$ at pleafure, and from $F$, the Center of the vanifhing Line, draw FE; then is FE the Diftance of that vanifhing
niming Line; therefore by making FL equal to the Diftance FE, and by drawing LI parallel to the Ray RS, we shall have I for the vanifhing Point of that Ray.

## Method 4.

When a Ray comes from behind the Spectator's Eye towards the Picture, in a Plane oblique with the Picture.
Fig. 98. Let IG be the vanifhing Line of that Plane, ZY a Ray of Light, and XYZ its Angle of Inclination.---From L, the tranfpofed Place of the Eye, draw LG parallel to the Ray ZY, which will give $G$ for its vanifhing Point.
COROLLARY.

From hence let us remember, that the Center C, or F, of a vanifhing Line EK, IG, of a Plane of Rays, will be the vanifhing Point of all Shadows which are cait by perpendicular Objects upon the Ground; becaufe that Point * muft be in the horizontal Line, and alfo in the vanifhing Line, of the Plane of Rays; fuch are the Points C and F .

To find the Shadow of an Object which is fuppofed to fland perpendicular to the Ground, when the Rays come in Planes parallel to the PiEture.
Fig. 99. Let FG be the Picture, AB the Reprefentation of a perpendicular Object whofe Shadow is fought; and let HL be a Ray of Light, whofe Inclination with the Ground is equal to the Angle HLC.---Through B, the Seat of the Object, draw Ea at pleafure, but parallel to the horizontal Line; and through A draw R a parallel to the Ray HL, cutting E a in a ; then is Ba the Shadow of BA.
Tig. 100 . Again, Let abcd be a perpendicular Plane, whofe vanifhing Point is C the Center of the Picture, and let HL be a Ray of Light.---Through the Seats $\mathrm{a}, \mathrm{b}$, of the Perpendiculars a d, b c, draw af, be, parallel to the horizontal Line, and through d, and c, draw Lf, Re, parallel to the Ray HL, cutting af, be, in $f$ and e ; finally, from f and e, draw fe, then is abef the Shadow of the Plane abcd, and fe continued will vanifh into $C$, the vanifhing Point of $a b$, and $c d$.

> Of SHADOWS by the SUN.

Now, when the Shadow of any perpendicular Object is produced by Rays which are fuppofed to come in Planes parallel to the Picture, that Shadow may be found by Calculation: Thus, when the Angle of Inclination is 45 Degrees, then the Shadow will be equal to the Height of the Object, as in the two laft Figures; therefore, by putting Unity for the Height of the Object, we may have the following Proportions, viz.

Angle of Inclination. Length of the Shadow. Deg. Min.

|  | 00 | No Shadow. |
| :---: | :---: | :---: |
| 78 | 45 | r-5th Part of the Object. |
| 67 | 30 | $2 .-5$ ths ditto. |
| 56 | 15 | 7-10ths ditto. |
| 45 | 00 | is The Height of ditto. |
| 33 | 45 | I 2-4the. once the Length of ditto and Half. |
| 22 | 30 | 2 4-9ths. twice the Len. of ditto and 4-9ths. |
| 11 | 15 | 5 Times the Length of ditto. |
| 00 | -0) | Infinite. |

From hence, then, we fee the Reafon why the Shadows produced by the Sun are very long in a Morning and Evening, and why they grow fhorter and fhorter the nearer the Sun approaches to the Meridian.

## The foregoing Rules applied to Practice.

To find the Shadow of an Object, when the Light comes from behind the Spectator towards the Picture.

Let $A B$ be the Picture, $C$ its Center, $C E$ its Diftance, $I K$ a Fig. 10 . Ray of Light, $D$ the vanifhing Point of the Rays of Light, abcd a perpendicular Plane whofe Shadow is fought, and $L$ the vanifhing Point of the Shadow which is caft upon the Ground by the perpendicular Sides ad, bc.

From a and b, the Seats of the Perpendiculars ad, bc, draw Lines to $L$, the vanifhing Point of the Shadow; and from $d$ and c, the Extremities of ad, bc, draw Lines to D, the vanifhing Point of the Rays; then from where they cut $a \mathrm{~L}$ and bL , draw ef, and then is abfe the Shadow of abcd; which if continued will vanifh into $C$, the vanifhing Point of $a b, c d$.

To find the Shadow of a perpendicular Object when the Light comes from behind the Picture.
Fig. rez. In this Figure, H is given for the vaniming Point of the Shadow, KI for a Ray of Light.---Draw FD parallel to IK, which will give D for the vanifhing Point of the Rays of Light; then from the Point H of the Shadow, draw Lines through all the lower Corners, a, e, f, of the Object, and continue them at pleafure ; then from D, the vanifhing Point of the 'Rays of Light, draw Lines through the upper Corners $b, c, d$; which will give the Points $\mathrm{g}, \mathrm{h}, \mathrm{i}$, from whence the Shadow a ghif, may be compleated.

In the two laft Figures, I have drawn out every Line and Point which is neceffary in the Work, and have alfo added the Angles IGK, IKL, for the Inclinations of the Rays, to make the Thing more intelligible.

Here let us obferve, that as the Shadow of every perpendicular Line, will vanifh into the vanifhing Point of the Shadow ; fo alfo the Shadow of every oblique Line, will vanifh into the vanifhing Point of that Line : Thus ag is the Shadow of the Perpendicular ab , and gh of the oblique Line cb ; and $\mathrm{gh}, \mathrm{cb}$, will both vanifh into $G$ : For fince the Shadow is caft upon a Plane perpendicular to the Object which projects it, therefore the Shadow.hg, and the Edge $c b$, are to be confidered as parallel, and confequently will tend to the fame vanifhing Point.

I have hitherto confidered Shadows as projected upon the Ground, and the Planes which project them as perpendicular to it; but by the fame Rules any other Shadows are to be determined, whether the Planes upon which they are caft are perpendicular, parallel, or oblique, or however the original Objeets are fituated : And therefore, thus much might have fufficed to explain the Theory and Practice of Shadows; fo far as is generally neceffary in a Picture ; but that this Part of Yerfpective may be made as familiar as poffible, I have added feveral ufeful Examples in the Practical Treatire, Book the Second.

XVIII .






## S ECT. II.

Of SHADOWS projected by the CANDLE, LAMP, Efo.

THE Rays of Light from a Candle may be conceived to flow from a fingle Point, like the feveral Radii of a Circle from its Center. The 94th Figure reprefents a Plane of thefe Rays, which is fuppofed to ftand perpendicular to the Ground Plane HP; where $L$ is the luminous Point, and $S$ its Seat upon the Ground.

Now fince thefe Kinds of Light are but feldom chofen for a Picture; and fince the Method for determining Shadows projected in this Manner, is extremely eafy; there needs but very little to be faid upon it: I fhall therefore treat this Section with the utmoft Brevity.

The firft Thing neceffary in order to determine the Shadow by a Candle, is, to give a luminous. Point, and its Seat in the Picture; then by drawing Lines from thofe Points through the Extremities of any Object, their mutual Interfections with each other will give the Appearance of that Shadow.

Thus, let $L$ be a luminous Point, S.its Seat, and abcd the Fig. 103 Reprefentation of a fquare Plane: From S and L draw Lines through the Extremities a, b, c, d, and their Interfections at e and $f$ will give the Shadow aefb.

Again, let L be a luminous Point, S its Seat, and ab, cd, ef, gh, Fig. 104: be the Reprefentations of feveral perpendicular Objects whofe Shadows are fought:

From S and L draw Lines through the Extremity of each Line, and the Points where they cut each other, will fhew the Length of the Shadows, as in the Figure.

## C H A P. VI.

Of the Diftance and Height of the Eye, of the Size of the Picture, and of the true Point of Sight, Ger. with Some Considerations upon the Appearance of circular Objects upon the Picture.

1. Of the Distance of the Eye.

THE choofing a proper Diftance for the Eye is fo effential in all Perfective Reprefentations, that without a nice Obfervance thereof every Object will appear unnatural and prepofterous, be the Rules by which it was drawn ever fo true in Theory, or fo exactly obferved in Practice. And the Reafon of this will appear extremely obvious, if we confider that there is a certain Diftance at which the Eye can fee an Object with more Diftinctnefs than in any other Point of View. Now, That Dirtance may be called the true Point of Sight in respect to That Object; and what is faid of one Object will hold equally true of any Number of Objects: And therefore, as it is the Bufinefs of Perfective to draw the Reprefentations of Objects, as they appear to the Eye, under the mort agreeable Shape, it follows, that the Diftance to be work'd with upon the Picture, fhould be chofen in fuch a Manner that each Reprefentation fall make the fame agreeable Figure to the Eye, as the Originals themfelves would do were they feel under the fame Angle.
Fig. 105 . To explain the Senfe of this more fully, let ADFK be a Pictore, C its Center, NL the horizontal Line, AB one Side of a geometrical Square parallel to the Picture ; and let it be required to find the Reprefentation of that Square as feen at the feveral Diftances CG, CI, CE.

From A and B draw Lines to C, the vanifhing Point of the oblique Sides; and from C feet off the Several Diftances of the Eye upon the horizontal line in the Points $\mathrm{O}, \mathrm{N}, \mathrm{H}$; then from there Points draw Lines to $B$, cutting $A C$ in the Points $a, d$, $e$ and from a, d, e, draw Lines parallel to $A B$ : Then foal we have the Reprefentation ABab as pen at the Diftance CE, the Reprefentaton $A B c d$ as feen at the Diftance $C I$, and the Reprefentation $A B f e$ as feen at the Distance CG.

## Of the Distancer of the Eye.

Now, by infpecting the Figure, we fhall find that the apparent Depth Ae, of the Reprefentation Abfe , which ought to be forefhorten'd, is longer than the parallel Side AB, fo that the Figure which fhould reprefent a Square, is a Parallelogram; and therefore this Reprefentation will not appear to be true: And if the Diftance be at I, then the Depth will be longer than it ought, becaufe the Figure ABcd will ftill look like a Parallelogram: But if the Diftance be taken at E , then the Reprefentation will appear of a more proper Depth; and therefore the Diftance CE is properer for a Picture of this Dimenfion. And if Lines are drawn from the feveral Points of View G, I, E, to the Extremities $N$ and L of the Picture, then thefe Lines will fhow the Angles under which the Picture is feen at thofe Diftances ; viz. that at $G$ will be an obtufe one, that at I a right one, and that at E an acute one: And therefore from hence we may conclude, that the Angle under which any Picture is to be feen, ought never to be fo great as a right one; and, by making an Experiment, we fhall find, that if it is much lefs than an Angle of 50 Degrees, the apparent Depths of fquare Objects will be too much forefhorten'd, by which Means thofe Objects which fhould reprefent fquare Bodies, will appear like fo many Parallelograms: However, in fome Cafes, fuch as in painting Deceptions for Gardens, or for Pictures with curvilinear Objects, the Diftance fhould be taken as great as poffible; which is left to the Difcretion of the Artift.

There are feveral other Reafons to be given for choofing a proper Diftance for the Eye; but as one Example is fufficient to fhew the Abfurdity and Inconveniency of difregarding, or not knowing, this effentialPart of Perfpective, it becomes needlefs to produce any others.

## 2. Of the Height of the Eye.

"IS the Height of the Eye that determines the Height of the horizontal Line from the Bottom of the Picture ; and therefore, it is that which gives the whole Space for the Reprefentation of the Ground. And in taking the Height of the Eye, we muft be careful not to let it be fo great as the Diftance of the Eye ; fince the fame bad Confequences will follow from thence as in choofing an improper Diftance. For let PP be a perpendicular Section of the Picture, AP an original Line perpendicular to it, and HE the Fig. 106. Height of the Eye.--Draw AE, and then will Pc be the Reprefentation of PA: But if the Eye be placed at I, fo as to be equal to its Diftance IP, then will the Reprefentation PC be too long in
the Picture ; and the nearer the Eye is brought to P, (fuppofe at Q) that is, the more the Height QR exceeds the Diftance QP, the more prepofterous will the Reprefentation P a appear. Indeed if any Fig. 10\% original Object, as AB , be parallel to the Picture, the Height of the Eye will have no Effect upon this kind of Reprefentations, provided the Eye moves in the fame perpendicular HI ; for the Reprefentation $a b$, is equal to the Reprefentation CP.
3. The Confequences of viewing Pictures from any ether than the true Point of. Sigbt.

FROM what has been faid upon the Diftance and Height of the Eye, it muft be manifeft, that no Perfpective Reprefentations will appear fo natural as when viewed from the true Point of Sight ; becaufe, at that Point, all the Rays which are fuppofed to come from the original Objects, and produce their feveral Projections upon the Picture, will concur at the Eye in their proper Point, and thereby exhibit a Picture upon the Retina exactly fimilar to that of their Originals.

But again, If the Eye is not placed in the true Point of Sight, the Projection of all Objects which are not parallel to the Picture, will not feem to tend to their proper vanifhing Points; and for that Reafon fuch Reprefentations will feem to ftart out of their proper Places, will lofe their juft Proportions, and confequently, will conwey a jumble of confufed Appearances to the Eye : And to this we may add alfo, the fhocking Effect it will have upon the horizontal Line in particular, which is always governed by the Place of the Eye.

What has been faid upon this Head, relates principally to Pictures painted upon uneven Grounds, fuch as Domes, vaulted Roofs, irregular Walls, $\mathcal{E}_{c} c$. where the leaft Variation from the true Point of Sight, will be productive of the above, and other bad Confequences: For as to flat Pictures, the Fancy will be ready to give fome Affiftance towards correcting what is not ftrictly right in them; and therefore, a little Variation of the Eye from the true Point of Sight, is allowable in fuch Cafes: For no great Inconveniency will appear, fo long as the Eye keeps upon a Level with the horizontal Line.
4. Of the Size of the Picture.

Rig. 1 Height of the Eye....Thus, let CE be the Diftance of the Eye, and CP its Height.
Of the SIzE of the Picture.

With the Diftance CE defcribe the Circle FKAD, and make CI equal to CP; through $P$ and I draw $A K, D F$, parallel to the horizontal Line, cutting the Circle in A, K, D, F; from which Points draw the Lines AD, FK: Then fhall we have a Square, which will give the utmoft Size a Picture fhould be of if feen from no greater Ditance than CE. But if the Height of the Eye be lefs than CP, then the Picture will be a Parallelogram, which is the moft general Shape given to Pictures.---This Method of limiting the Size of the Picture to the Diftance and Height of the Eye, will be of great Ufe in feveral Operations.

## 5. Some Confiderations upon the Appearance of round Objects upon the Picture.

FROM what has been faid upon the Diftance of the Eye, $\mathcal{E} c_{0}$ it may feem very improbable that any perfpective Reprefentation fhould have a difagreeable Effect, if the Rules we have laid down be nicely obferved: Yet there are fome Cafes, perhaps, in which the Artift will think it better to be guided by his own Judgment, than to follow the ftrict Rules of Perfpective. This feems to have been the Opinion of Monfieur Frefnoy: For in his excellent Poem upon Painting, tranllated by Mr:Dryden, he fays, "Though
"Perfpective cannot be called a perfect Rule for Defigning, yet it "s is a great Succour to Art, and facilitates the Difpatch of the "Work; tho, frequently falling into Error, it makes us behold "Things under a falfe Afpect; for Bodies are not always repre"fented according to the Geometrical Plane, but fuch as they ap"pear to the Sight." But as there are different Opinions upon this Subject, I fhall beg Leave to offer my Thoughts upon it.

Suppofe it was required to draw the Reprefentation of a Range of Columns parallel to the Picture; if they are drawn according to the ftrict Rules of Perfpective, then that Column which is in the Center of the Picture will be the leaft, and confequently, thofe on each Side of it will be larger and larger continually, the farther they are removed from the Center of the Picture. But to explain this more fully: Let KLMN be a Plane which paffes through the Eye Fig. ros. parallel to the Ground; then will PP be the horizontal Line, and C the Center of the Picture: And let AB, H, I, be three Columns cut by this Plane; then let Lines be drawn from the Extremity of each Circle to the Eye; and the Sections ab, cd, ef, with the
the Picture, are the Projections of thofe Circles upn the Picture; and by meafuring the feveral Reprefentations we fhill find, that $\mathrm{cd}_{3}$, and ef, are much longer than $a b$. From whence ne may conceive, that the farther any Column is removed from the Center of the Picture, the larger will be its Reprefentation; and we may moreover conceive, that this Increafe of the apparent Mugnitude of the Columns, is owing to the Obliquity of the Lines $\mathrm{gh}, \mathrm{ik}$, with the Picture, which Lines meafure their apparent Widths. Now the Queftion is, Whether Columns fituated in this Minner are to be thus reprefented upon the Picture, or not?

The Definition I have given of the Word Perfpective, is this; viz. To draw the Reprefentations of Objects as they appear to the Eye, $\mathcal{E}_{c}$. and I have avoided the more general Definition, viz. of drawing the Reprefentation of Objects by the Rules of Georretry, $\mathcal{E}$. as the former appeared to be more fignificant of what I intended to exprefs by the Term Perfpective. For fince the Fallaces of Vifion are fo many and great*, and fince we form our commor Judgment and Eftimation of the Appearance of Objects from Cutom and Experience $\uparrow$, and not from mathematical Reafoning; therefore it feems reafon-

[^16]
## Some necaffary Obfervations, EDC.

reafonable not to comply with the ftrict Rules of Mathematical Perfpective in fome particular Cafes (as in this before us) but to draw the Reprejentation of Objects as they appear to the Eye; and therefore, I preiume, a Painter fhould reprefent thofe few Objects which are an Exception to the General Rules of Perfpective, in fuch a Manner as may not offend the Eye of any common Spectator. For if the above Columns are to be reprefented according to the ftrict Rules of this Art; then the Columns as they recede from the Center of thie Picture will grow thick and clumfy, their Intercolumnations will be continually growing lefs and lefs, and the whole Beauty of the Building will be intirely deftroyed. *
" of a convex Figure, and an uniform Colour; when the Idea we receive from thence, is
"only a Plane varioully colour'd, as is evident in Painting. To which Purpofe I fhall here
" infert a Problem of that very ingenious and fudious Promoter of real Knowledge, the lear-
" ned and worthy Mr. Molineux; and it is this: Suppofe a Man born blind, and now adult,
" and taught by his Touch to diftinguifh between a Cube and a Sphere of the fame Metal,
" and nearly of the fame Bignefs, fo as to tell when he felt one and t'other, which is the
"Cube, which the Sphere. Suppofe then the Cube and Sphere placed on a Table, and the
" blind Man be made to fee: Quare, Whether by his Sight, before he touched them, he
"could now diffinguin, and tell, which was the Globe, which the Cube. To which the
" acute and judicious Propofer anfwers : Not. For though he has obtained the Experience
" of, how a Globe, how a Cube, affects his Touch; yet he has not yet attained the Ex-
"perience, that what affects his'Touch fo or fo, muft affect his Sight fo or fo: Or that a
"p protuberant Angle in the Cube, that prefied his Hand unequally, fhall appear to his Eye as
" it does in the Cube. I agree with this thinking Gentleman, whom I am proud to call my
"Friend, in his Anfwer to this his Problem; and am of Opinion, that the blind Man, at
or. firt Sight, would not be able with Certainty to fay, which was the Globe, which the Cube,
" whilft he only faw them, though he could unerringly name them by his Touch, and cer-
" tainly diftinguifh them by the Difference of their Figures felt. This I have fet down, and
" leave with my Reader, as an Occafion for him to confider how much he may be beholden
" to Experience, Improvement, and acquired Notions, where, he thinks, he has not the
" leaft Ufe of, or Help from them." Vide Locke's EJay upon Human Underfanding, Vol. I, Ch. 9. But this is no new Opinion; for fo old an Author as Lucretius, takes particular Notice of it ; for he, after having given innumerable Infances of the Errors in our Judgment, in regard to Sight, fums them up in the following Lires.

Catera de genere boc mirando multa videmus,
Que violare fidem quafi Senfibus omnia quertunt:
Nequicquam. Quoniam pars borum maxima fallit
Propter opinatus Animi, quos addimus ipfj,
Pro vifis ut fint, quae non funt fenfibu' vifa. Lucret. Lib. 4.

> Or, as Mr. Creech has tranlated it:
" Ten thoufand fuch appear, ten thoufand Foes
"To Certainty of Senfe, and all oppofe :
"In vain, 'tis Judgment, not the Senfe miftakes,
"Which fancy'd Things for real Objects takes." Vide Creecb's Lucret. B4.

[^17]What has been faid upon this Subject, relates principally to round or cylindrical Bodies, fuch as Globes, Columns, or the like; but as to angular ones, (efpecially thofe that are Square) fince their apparent Widths are perpetually increafed the more diagonally they are feen by the Eye, therefore, the Reprefentations of fuch Objects upon the Picture fhould continually grow larger and larger in Width the more they are removed from the Center of the Picture. Thus the Reprefentation of the Square Q, which is feen only in Front, cannot appear fo large as the Reprefentation of the Square R; which is viewed as a Triangle. I fay, that the apparent Magnitude of Objects that are Square or Triangular, will be greater when view'd Angle-wife, than when feen in Front: But the apparent Magnitude of Columns, or any other round Objects, will always be the fame at the fame Diftance; becaufe, in the firf Cafe, the Diagonal of a Square (which in fome Views meafures its apparent Width) is longer than its Sides; but in the latter Cafe, the Diameter of a Circle (which conftantly meafures its apparentWidth) is always of the fame Length; and therefore to reprefent Columns, $\mathcal{E}_{c}$. larger and larger, when they are at a greater and greater Diftance, is, I prefume, falfe in Theory, (I mean in an optical Senfe only) and cannot be true in Practice. To this it may be faid; Why then fhould they not be reprefented lefs and lefs in proportion to their feveral Diftances, fince in fact they are fo ? To which I anfwer again, that by a Habit of judging, and from the prevailing force: of Experience, we are taught to think, they are all of the fame: Size, becaufe they are upon the fame Parallel with the Eye. Thus, for Inftance; when we ftand before the Middle of a Building of any confiderable Length, we apprehend the Ends to appear exactly as high as the Middle of it, though in fact they cannot, becaufe the Angle fubtended at the Eye from the Middle, is greater than thofe fubtended at the Corners. Again, fuppofe it was required to draw the Reprefentation of round Balls, or Globes, which are fuppofed to be at the fame Diftance from the Picture, according to the ftrict Rules of Mathematical Projection : Then the Projection of that Ball only which is in the Eye's Axis will be a Circle, and, being properly fhaded, will appear like a Globe ; but all the other Projections, which are not in the Eye's Axis, will be Elliptical, and, fhade them how you will, they can never appear like Globes to any common Spectator: I fay to any common Spectator, becaule fuch Appearances contradict the common Idea which Men in general have form'd to themfelves of Rotundity. In fhort, Perfpective,

In a frictly Mathematical or Optical Senfe, is one Thing; and Perfective, according to the Acceptation of that Word among Painters, is another: The Firft teaches how to defcribe on a Plane, to a mathematical Exactnefs, the Projections of any Objects; but the Second, like a modeft and judicious Mafter, teaches the moft fimple and general Principles of Art; and inftead of leading us into the Mazes of Lines and Angles, and lofing us in the Labyrinths of mathematical Reafoning, directs us only to the Study of Simpiecity, which is the Foundation of Grace and Beauty.

I know it may be faid, that if we make choice of a proper Diftance, all Inconveniencies of this Kind may be avoided: But let the Diftance be ever fo proper, yet ftill the Projections of Coumns, Ec. as they are removed farther and farther from the Center of the Picture, will grow larger and larger continually; which furely ought not to be admitted.

Thefe are the Reafons which induced me to confider this Subject in a particular Manner; but whether they are fufficient, or not, to anfwer the intended Purpofe, is fubmitted to the Candour of every ingenuous Reader.

## CHAP. VII.

## Of Aerial Perspective, Chiara Oscuro ${ }_{2}$ and Keeping in Pictures.

From Mr. Hamileono

BY Arrial Perspective is meant, the Art of giving a due Diminution or Degradation to the Strengh of Light, Shade, and Colours of Objects, according to their diffe" rent Diftances, the Quantity of Light which falls upon them, " and the Medium through which they are feen.
"The Chiara Oscuro confifts more particulaly in * ex" preffing the different Degrees of Light, Shade, ant Colour of
"Bodies, arifing from their own Shape, and the Poffton of their
"Parts with refpect to the Eye and neighbouring Objects, where-
"" by their Light or Colours are affected.
"And Keeping, is the Obfervance of a due Proportion in " the general Light and Colouring of the whole Pictu:e; that no "Light or Colour in one Part, may be too bright or ftrong for " another; but that a proper Harmony amongft them all together " may be preferved.
"All thefe are neceffary Requifites to a good Picture, and may " be properly enough included within the general Name of Aerial "Perfpective, as they all relate to the different Degrees of Strength " of the Light and Colouring, according to the Circumftances of " the Shape and Pofition of the Objects with regard to each other, " the Eye, and the Light which illuminates them.
"The Eye does not judge of the Diftance of Objects barely by
" their apparent Size, but alfo by their Strength of Colour and
" Diffinction of Parts; it is not, therefore, fufficient to give an
"Object its due apparent Bulk, according to the Rules of Perfpec-
" tive, unlefs at the fame Time it be expreffed with that proper
"Faintnefs and Degradation of Colour which that Diftance
" requires.
" Thus, if the Figure of a Man at a Diftance were painted of " a due Size for the Place, but with too great a Diftinction of

[^18]"Parts, or too ftrong Colours, it will appear to ftand forward, and feem proportionably lefs, to as to reprefent a Dwarf fituated nearer the Eye, and out of the Plane on which the Painter intended he fhould ftand.
"By the Original Col our of an Object, is meant that Colour: which it exhibits to the Eye when direetly expofed to it in a full, open, uniform Light, and at fuch a moderate fmall Diftance as to be clearly and diftinctly feen.
"This Colour receives an Alteration from many Caufes, the principal of which are thefe :
"1. From the Object's being removed to a greater Ditance from the Eye, whereby the Rays of Light which it reflects are lefs vivid, and the Colour becomes more diluted, and tinged in fome meafure with the faint blueifh Caft, or with the Dimnefs or Hazinefs of the Body of Air through which the Rays pafs.
" 2. From the greater or lefs Degree of Light with which the Object is enlightened: The fame Original Colour having a different Appearance in the Shade from what it has in the Light, although at an equal Diftance from the Eye, and fo in Proportion as the Light or Shade is fronger.
"3. From the Colour of the Light itfelf which falls upon it, whether it be by the Reflection of coloured Light from any neighbouring Object, or by its Paffage through a coloured Medium; which will exhibit a Colour compounded of the Original Colour of the Object, and the other accidental Colours which the Light brings with it.
"4. From the Pofition of the Surface of the Object, or of its feveral Parts with refpect to the Eye; fuch Parts of it as are directly expofed to the Eye appearing more lively and diftinct than thofe which are feen flanting.
"5. From the Clofenefs or Opennefs of the Place where the Object is fituated, the Light being much more varioufly directed and reflected within a Room, than abroad in the open Air; every Aperture in a Room giving an Inlet to a different Stream of Light with its own peculiar Direction, whereby Bodies in fuch a Situation will be very differently affected with refpect to their Light, Shade, and Colours, from what they would be in an open Place.
"6. Some Original Colours naturally reflect Light in a greater is Proportion than others, though equally expofed to the fame $\mathrm{K}_{2}$ " Degrees
"Degrees of it; whereby their Degradation at fevera Diftances " will be different from that of other Colours which reflect lefs
" Light.
"From thefe feveral Caufes it arifes, that the Colorrs of Ob«\% jects are feldom feen pure and unmixed, but generally arrive at
" the Eye broken and foftened by each other; and tlerefore, in
"Painting, where the natural Appearances of Object are to be
" defrribed, all hard or fharp Colouring ought to be awided.
"A Painter, therefore, who would fucceed in Aerall Perfpec-
"tive, ought carefully to ftudy the Effects which Liftance, or different Degrees or Colours of Light, have on eacl particular Original Colour, to know how its Hew or Strengthis changed " in the feveral Circumftances above-mentioned, and o reprefent " it accordingly; fo that in a Picture of various-colourd Objects
" he may be able to give each Original Colour its own proper Di" minution or Degradation according to its Place.
"Now, as all Objents in a Picture take their Meafurs in Pro" portion to thofe placed in the Front, fo, in Aerial Perfpective,
${ }^{*}$ the Strength of Light, and the Brightnefs of the Colcurs of Ob" jects clofe to the Picture, muft ferve as a Meafure, vith refpect " to which, all the fame Colours at feveral Diftances, muft have " a proportional Degradation in like Circumftances. But, as in " Mufick, it is not neceffary to the Harmony, that the Inftru" ments fhould be tuned to the Concert Pitch, but they may be " fet above or below it, fo long as they are in tune to each other ; " fo in Painting, it is not requifite that the Meafures on the in" terfecting Line of the Picture, or the Brightnefs of the Light " there, fhould be equal to the Life; but they may be taken " greater or lefs, fo long as every Thing elfe in the Picture bears. " a true Proportion to that which is chofen as the firft Standard. "Hence, almoft any Degree of Light may be taken for the ev greater Light in a Picture, when the leffer Degrees of Light are « expreffed with darker or weaker Colours; for any Degree of " Light may either reprefent a Light in refpect of a darker, or it " may ferve as a Shade to a lighter; and it matters not in Point " of Keeping how light or how dark a Picture is in general, fo " that its feveral Parts have proportionable Degrees of Light and
"Shade given them.
"In order, therefore, to the giving any Colour its due Dimi-
"nution in Proportion to its Diftance, it ought to be known, what

## Of Aelial Perspective, Chiara Oscuro, ©G.

"the Appeannce of that Colour would be, were it clofe to the "Picture, Regard being had to that Degree of Light which is "chofen as tie principal Light of the Picture; as in order to the " giving any Object its due apparent Size, its true Size muft be " reduced to the fame Scale with the Meafure on the Bottom of " the Picture.
"For if any Colour fhould be made too bright for another, or " for the general Colours employed in the reft of the Picture, it " will appear too glaring, and feem to ftart out of its Place, and "throw a Flatnefs and Damp on the reft of the Work ; or, as the " Painters exprefs it, the Brightnefs of that Colour will kill the "reft:
"No Painting can exprefs the dazzling Brightnefs of the Sun, * or even its reflected Light coming from polifhed Metals, with "that fparkling Vivacity as it appears in the Camera Obfoura, in " the Images of polifhed Surfaces on which the Sun fhines; or if " it could in fome Sort be imitated in a Picture, by the Affiffance " of Gilding, it would not have a good Effect with regard to the " other Colours, which it would too much outhine ; and thereby " hurt the Keeping: And this is one Defect which the Reprefen"tation of Objects in the Camera Obfura is liable to; for by " reafon of the Refraction of the Rays by the Glafs, thofe Objects " which naturally reflect lefs Light, lofe a greater Proportion of " it than thofe which reflect Light more plentifully; whereby the "due Keeping in the whole, is not fo exactly preferved as in " direct Vifion, the Lights and Shades appearing generally too " ftrong for each other."
Thus far Mr. Hamilton. And I have thought proper to add the following Figure, with a Defign of fixing what he has faid upon the Subject more ftrongly in the Memory.

Let E be the Eye, PP the Picture, RS, AB, CD, EF, and HL, the fame Object feen by the Eye at different Diftances; now, the farther they are removed from the Eye, the larger will be the Space of Air through which they are feen, and the more they will be tinged with its Hew ; therefore, in Proportion as the Reprefentations $\mathrm{Ps}, \mathrm{Pb}, \mathcal{E}_{\mathrm{O}} c$ are more and more diminifhed upon the Picture, fo likewife, in the fame Proportion, muft the Original Colour of thofe Objects be more and more broken and diluted with the Colour of the Air. Thus, fuppofe the Reprefentation Ps of $R S$, to be painted of the Original Colour ; then, becaufe the Reprefentations $\mathrm{Pb}, \mathrm{Pd}, \mathrm{Pf}$, and Pl , are perpetually diminifhed in in colouring thofe feveral Reprefentations, Care muft be taken to diminifh that alfo in the fame Proportion.

There may be fome Exceptions made to this general Rule by a nice Obferver of Nature, for there are fome Incidents will happen which feem to contradict it; fuch as a white Houfe, or any other very light Object directly oppofed to the Sun at a great Diftance; yet notwithftanding, what has been advanced will be of great Service in fixing right Ideas of thefe effential Requifities: And I may venture to affirm, that without a general Obfervance thereof, every Picture will be at beft but a flat and lifelefs Performance.

I might now proceed to the Confideration of fome other Things relative to Perfpective; but fince they are not at all effential in the Theoretical Part, and as I muft take Notice of them in anothers Place, I fhall therefore put an End to this Book

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XXII.



## THE

## PRACTICE

## Of

# PERSPECTIVE: 

 BeingThe SECOND BOOK

of
Dr. BROOK TAYLOR's Method of Perspective made eafy, छoc.
By JOSHUA KIRBY, Painter.

The Practice [of Painting] ougbt always to be built on a rational Theory, of which Perspective is both the Guide and the Gate, and without which it is impofible to fucceed either in Defigning, or in any of the Arts depending thereon.

Leonardo da Vinci upon Painting, p. $3^{6}$.

The SECOND EDITION.

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## TO THE

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## PAINTING, SCULPTURE, ARCHITECTURE, \&c in LONDON.

Gen themen,

AS Perspective is abfolutely neceffary to a Just Design, give me Leave to dedicate this Book to You on the Subject, It is the Product of many Years Application and Study, and wrote with an Intention to render that hitherto perplexed, but ufeful Art, eafy and familiar. How I have fucceeded in the Attempt, is fubmitted to your Candour and Judgment ; and I hope that this Dedication will be received as an Inftance of my Gratitude, for the Favour of that Encouragement and Recommendation, which you have been pleafed to give to the Work.

I do not prefume to offer any Thing new to the Principal Members of the Society; for I am Book II.

A
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## DEDIGATION.

not fo vain as to think I can give any Infructions to Perfons of fuch fuperior Abilities: But if I can contribute a little towards inftructing the Pupils in the firft Rudiments of Defign, it may fpare fome Time and Trouble, and I hope will be accepted as a Token of my Regard for You, and Affection for thofe Arts.----I thall only add, that it is my fincereft Wifh, that every Encouragement may be given to your indefatigable Endeavours, in promoting the ARTs of Painting, Sculpture, Architecture, $छ^{\circ} c$. That the Pupils may do Honour to their feveral Masters, and become Ornaments to their Country; and that every other Advantage may concur to raife the Glory of the English Academy to the higheft Pitch.

$$
1 \mathrm{am}
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Gentlemen,

## Vour moft Obliged,

## Humble Servant,

JOSHUA KIRBY.

## P R E F A C E.

IN this Practical Book upon Perspective I hall endeavour to give fome general Methods for finding the Reprefentations of all Kinds of Objects, bowever they are fituated in regard to the Eye or the. Picture, or bowever irregular they are among themfelves. And fince great Care bas been taken to adapt every Example in this Part to the Theory, the Reader may be fatisfed that every Figure is Arictly true, and capable of a Mathematical Demonftration; So that thofe whom Curiofity will not invite, or Leifure permit, to go regularly through the preceeding Theory, need not trouble themfelves about it, becaufe what follows will be fufficient for their Purpofe. But let them confider, that it is in this as in all other Studies, with which, if a Perfon defires eitber to be thorougbly acquainted, or to profit by bis Study, be muft read with Attention, draw out every Figure as be proceeds, and be well acquainted with one Example before be begins with another.

And fince it is prefumed that every Example in the following Work, may be as eafily underfood and applied to Practice by every Student in the Arts of Defign, as are the common Principles of Aritbmetick by every ordinary Mechanick; therefore it is boped that Perspective will be no longer tbougbt an abftrufe and difficult Study, nor be difregarded as. trifing and infignificant; but that the young Tyroes in the above Arts will firft make Pers Pective familiar to them, and treat ber woith the Refpect which foe deferves, as the PARENT of the noble Art of Painting; and upon whofe general, though not rigid Precepts, every Defign muft be regulated, if the Artit intends it Jball appear a true Reprefentation of Nature.

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## PRACTICE of PERSPECTIVE made EASY, $\overbrace{}^{2} c$.

 BOOK: CHAPS.


## An Introduction to the PRACTICE of PerSpective.

IN order to convey a general Idea of Perspective with as much Eafe as the Nature of the Thing will admit, let ABCD Fig. $\mathrm{x}_{0}$ reprefent a fquare Board ftanding perpendicularly upon the Ground, which is reprefented by HM, and fuppofe the Figure EH to be looking at it; then it will be evident, that he muff fee it by means of an infinite Number of Rays of Light, which are continually reflected from every Point of the raid Object to his Eye. But fince the Rays which come from the four Corners only will be fufficient for our Purpofe, we will fuppofe that he fees it by means of the four Lines $A E, B E, C E$ and $D E$, which reprefont thole four Rays of Light: And, if we fuppofe a tranfparent Plane, like a large Piece of Glass, to be placed between the Object ABCD and the Spectator's Eye, it muff be obvious, that this Plane will cut the Rays of Light in their. Paffage to his Eye. Now, the Shape abed, which that cutting of the Picture makes with the Rays, is called the Projection of the real Object ABCD, upon the Glafs. And if inftead of this tranfparent Plane, we fuppofe GLOP to be a Canvas, and the above Projection to be drawn upon it in the very fame Manner as it was projected upon the Glass, then the Figure fo defcribed is called the Perspective Appearance of the real Object ABCD; for the Rays of Light coming to the Eye from the Points a, b, c, d, which are drawn upon the Picture, in the very fame Manner as they do from the corresponding original Points $A, B, C, D$, of the real Figure; therefore, if they are painted with the
the fame Strength of Colour, $\mathcal{F}$ c. they will to the Spictator $E H$, appear like that original Object: So that the whole Art of Perfpective confifts, in determining thefe, and the like Appearances, upon the Picture, and in giving them their proper Force and Colour.

Now let us obferve, Firft, If the original Figure be parallel to the Picture, then its Reprefentation will be exactly like it ; thus, ABCD is parallel to the Picture, therefore, a bcd, its Reprefentation, is exactly like it. For which Reafon, the Reprefentations of the Sides of all Objects that are parallel to the Picture, will not tend to any Points upon the Picture, but will be parallel amongft themfelves, and only proportionally, diminifhed as their Diftance from the Eye is greater or lefs: But the Reprefentations of the Sides of all Objects that are not parallel to the Picture, will vanifh into various Points upon the Picture, which, are therefore called, the vanifbing Points of fuch Objects.
Fig. 2. Seconidly. The Projections ABcd and efgh, of the Squares ABCD, EFGH, which lie flat upon the Ground, HM, will to the Spectator, EH , be the perfpective Appearance of thofe Objects : And the Reprefentations of the Sides $A B, c d$, eh and fg, will be: parallel to the Bottom of the Picture, but will be feverally diminifhed in proportion to their Diftance from the Picture. Thus AE is even with the Bottom of the Picture, and therefore its Reprefentation is the fame as the original Line AB, and confequently equal to it: But the Reprefentations cd , eh, and fg , will be perpetually diminifhed in the Degree of Diftance they are from the Picture ; as is evident by infpecting the Figure. For which Reafon, the Reprefentations $A d, B c$, ef, and $g h$, of Lines AD, BC, EF, and GH, which lie flat upon the Ground, and are parallel amongft themfelves, but not parallel to the Picture, will continually approach towards each other, 'till they vanifh into a Point C, exactly as high above the Bottom GL of the Picture, as the Eye is removed above the Ground HM. This, FC is equal to the Height of the Eye EH; and the Sides Ad, Bc, §?. will vanifh into the Point C. From hence then, we fee the Reafon why the Reprefentations of Objects are more and more diminiftied upon the Picture, the farther thofe Objects are fuppofed to be from it.

Tbirdly, We have obferved that the Reprefentations Ad and BC, of the oblique Sides $A D$ and $B C$ of the real Object, will vanifh into the Point C upon the Picture; and therefore the Point C may very properly be called the vanifhing Point of the Lines AD and

BC.----Now, in order to determine the vanifhing Point of any Line, we muft always draw a Line from the Eye parallel to that Line: Thus EC is parallel to AD , or BC , and therefore, C , where EC cats the Picture, is the vanifhing Point of AB , or BC .

And in like Manner, EJ being drawn parallel to the Line BK, Fig. s. which is ohlique with the Ground, will give $J$ for its vanifhing Point upon the Picture. For, from K draw the Ray KE to the Eye, which will cut the Picture in k ; then is k the Reprefentation of $K$, and $b$ is the Reprefentation of $B$; therefore, $k b$ is the Reprefentation of KB: And, if kb was continued upwards upon the Picture, it would cut E J in the Point J, and therefore J is its vanifhing Point.

Fourthly, If thro the vanifhing Point C, a Line HL be drawn parallel to the Bottom of the Picture, then that Line will be the vanifhing Line of all Objects that lie flat upon the Ground, or are parallel to it. Now this Line, HL, hath always been called the Horizontal Line, and therefore, I fhall call it by that Name in the following Work. Indeed, it is the moft ufeful of all vanifhing Lines; but neverthelefs, too much Strefs hath been laid upon it by almoft all Writers upon this Subject; who have paid no Sort of Regard to any other vanifhing Lines. But had they confider'd, that there are feveral Objects whofe Reprefentations cannot be correctly determined upon the Picture, without a general Knowledge of all Kinds of vanifhing Lines and vanifing Points, they would not have confined themfelves to the Horizontal Line only; and had they built their feveral Syftems of Perfeective upon as folid Principles as Dr. Taylor or Mr. Hamilton, their Works would not have been crouded with fuch a Confufion of Lines, nor with fuch a Number of ufelefs Examples; but they would have been more true, fimple, and of more general Ufe. But to return from this Digreffion.

Fifthly, Since the Horizontal Line is level with the Eye and parallel to the Ground, and, for that Reafon, the vanifhing Line of all Objects which lie flat upon the Ground, or are parallel to it; fo, for the very fame Reafon, the vanifhing Line of any other Object will be parallel to that Object. Thus, fuppofe ABK to be a triangular Plane, which ftands upon the Edge AK, perpendicular to the Ground: Then the vanifhing Line of this perpendicular Plane will be perpendicular to the Horizontal Line ; and if this Plane be perpendicular to the Picture alfo, then its vanifhing Line will pafs through the Center of the Picture; thus JE is the vanifhing Line of ABK.

Thus much, I prefume, may fuffice to give the unlearned Reader a tolerable Idea of Perfpective.-.-We will now give an Explanation of a few Terms made ufe of in the following Work, and then proceed to the Mechanical Part of Perfpective.

## DEFINITIONS.

1. 工HE Point of Sigbt, is that Point where the Spectator's Eye of Sight.
2. If from the Point of Sight E, a Line EC is drawn from the Eye perpendicular to the Picture, then the Point C, where that Line cuts the Hicture, is called the Center of the Pieture.
3. The Diftance of the Picture, is the Length of the Line EC, which is drawn from the Eye perpendicular to the Picture.
4. If from the Point of Sight $E$, a. Line $E C$ be drawn perpendicular to any vanifhing Line HL , or JF , then the Point C , where that Line cuts the vanifhing Line, is called the Center of that vanifing Line.
5. The Diftance of a vanibing Line, is the Length of the Line EC, which is drawn from the Eye perpendicular to the faid Line: And if PO was a vanifhing Line, then EJ will be the Diftance of that Line.
6. The Diftance of a vanifing Point, is the Length of a Line drawn from the Eye to that Point: Thus, EC is the Diftance of the vanifhing Point C , and EJ is the Diftance of the vanifhing Point J .
7. By Original Object, is meant the real Object whofe Reprefentation is fought: And by Original Plane, is meant that Plane upon which the real Object is fituated: Thus, the Ground HM is the Original Plane of ABCD, Ȩc. Fig. $1,2$.

## AXIOMS.

Fig. 2: 1. The Reprefentations of all Lines that are parallel to each other, but not parallel to the Picture, will have the fame vaniming Point: Thus, the Reprefentations Ad, Bc, of the parallel Lines $\mathrm{AD}, \mathrm{BC}$, have the fame vanifhing Point C .
2. The Reprefentations of all Lines that are parallel to each other, and parallel to the Picture alfo, will not vanifh into any Point upon the Picture, but will be parallel to each other: Thus, $A B, c d$, eh, and $f g$, are parallel to each other, becaufe the Originals $A B, C D, E H$, and $F G$, of thofe Lines, are parallel to each other.


## f. $\quad$ CHAP. II.

## Practicae Perspective.

SECT. 1 .
To prepare the Picture.---I. Of the Size of the Picture. 2. Of the Height of the Eye. 3. Of the Distance of the Eye.

'THE Size of the Picture muft be adapted ta the Diftance of the Eye, if it be an immoveable Picture, like the Side of a Room, Ceiling, or the like; which may very eafily be done by means of Frames, or other Compartments: But, if the Picture be an Eafel-Piece, * then the Bignefs of it may be left to the Artif's Difcretion. The Height of the Eye, muft always govern the Height of the Horizontal Line from the Bottom of the Picture ; and particular Care muft be taken not to let it be fo great as the Diftance of the Eye, fince it will be productive of very bad Confequences. And great Regard muft be had to the choofing a proper Diftance to be worked with; for otherwife, every Perfpective Reprefentation will have a very bad Effect.

## 2. Of the Height of the Eye.

Suppofe GLTP to be a Canvas, reprefenting the Size of the Fig, 3t Picture.----Divide the Bottom of it, GL, into two equal Parts in F, and draw FI perpendicular to the Bottom GL; then from F fet off FC, equal to the Height of the Eye from the Ground, and draw HL, through the Point C, parallel to the Bottom GL : And then will HL be the Horizontal Line, and C the Center of the Picture. Now, though the Height of the Eye in Eafel-Pictures is left intirely to the Difcretion of the Artift; yet, in general, low. Horizons have a much better Effect than high ones; for which Reafon, the Height of the Horizontal Line thould never exceed one half of the Height of the Picture; and, I believe, a little. Experience will teach any one, that if it is made equal only to one third Part of the Height of the Picture, it will be the moft proper Height of any : I mean only in regard to Eafel-Pieces; far if the

[^19]Picture be a fixed one, then the Height of the Horizonal Line mult be exactly level with the Spectator's Eye.
3. Of the DTSTANCE of the EYE.

The choofing a proper Diftance for the Eye is fuch an effential Requifite, that without a nice Obfervance thereof, every Perfpective Reprefentation will appear a fhocking Deformity; therefore, twe fhall be the more particular in fettling a proper Diftance for the Eye; the Neceffity of which will appear by the following Example.

In the $4^{\text {th }}$ Figure $I$ fuppofe $A B E D$ a real Square upon the Ground, and CE one Diftance of the Eye, and CI another. Now, by putting this Square into Perfpective, agreeable to thofe different Diftances, we thall have abED for the Reprefentation of the Square as feen at the Diftance CE, and EDCd for the Reprefentation of the Square as feen at the Diftance CI; and by infpecting the Figure, we may perceive, that the Reprefentation EDcd, which is feen by the Eye at the Diftance CI, does not appear like 2 Square, but looks much longer than tis wide, and therefore, it is a falfe Reprefentation; but the Reprefentation abED, which is feen by the Eye at the Diftance CE, has a more agreeable Appearance, and looks like a Square feen in Perfpective, and therefore is a more juf Reprefentation. Now, that this Difference between the two Reprefentations of the fame Object is wholly owing to the different Diftances of the Eye, is apparent from the Figure; and therefore, this one Inftance, out of many, may fuffice to fhew the Neceffity of choofing a proper Diftance to be worked with : In order to do which, the following Method feems the moft eafy, and the mof ufeful, of any I can think of.
Rigo 3.
Having drawn the horizontal Line HL, and fixed the Center C, of the Picture; draw a Line (as CP ) from the Center C , to one of the fartheft Corners (as P) of the Picture; draw alfo the Perpendicular CD, and continue it at pleafure; then from C, fet off the Length CP, upon CE, and call CE the leaft Diftance: Again, from $C$, fet off $C D$, upon the Line $C D$, equal to the longeft $\mathrm{Di}-$ menfions of the Picture, and call CD the greateft Diftance. That is, never let the Diftance you work with be greater than CD, nor lefs than CP; becaufe, as was obferved before, if the Diftance be lefs than CE, the Reprefentations will be too deep; and if it be more than CD, the Reprefentations will not be deep enough; and, I think, if a Mediun between thofe two Diftances be taken as a
general Rulf, it will produce the moft agreeable Shape of any Diftance whatfoever. Thus, the Reprefentation ED mn, (Fig. 4.). is determined by fuch a Diftance.

In this Place it may not be improper to take Notice; that the Diftance of the Picture is fometimes placed upon a Line as CD, perpendicular to the horizontal Line, and fometimes upon the horizontal Line itfelf; as the Nature of the Work may require. And I will alfo obferve, that the Diftance generally made ufe of in this Work, is the leaft Diftance, for the Conveniency of having as many Figures upon each Plate as was poffible. And the Reader is defired to remember, that the Letters CE will always ftand for the Diftance of the Eye, E for the Eye, or Point of Sight, C for the Center of the Picture, HL for the horizontal Line, and GL for the Ground Line, or Bottom of the Picture. For, to avoid Prolixity, I fhall not mention either of thofe Terms but upon fome particular Occafion. And that he may fix them the eafier in his Memory, I have made every Letter as fignificant as poffible: Thus E is the Eye, C the Center, HL the horizontal Line, $\mathcal{E}^{c}$ c. And I fhall, moreover, always fuppofe a Picture, as GLTP, (which may be confidered as a large Picture in Miniature) to be laid flat, and that we are actually at work upon it, in determining the Reprefentation of the following Figures.
S E C T. II.

Of OBJECTS which lie flat upon the GROUND, or that are in Planes perpendicular to the Picture.

1. To find the Reprefentation of a Point upon the Picture, after: baving prepared the Picture as above directed. Method i. By one danibing Point only.

LET A be the Point upon the Ground.----From A draw any Line at Pleafure, as A 1 , cutting the Bottom of the Picture in I ; and from the Eye E, draw EL parallel to A I, cutting the horizontal Line in $L$; then is $L$ the vanifhing Point of Ar; therefore, draw the Line LI, then from the Point A, draw a Line to E, cutting $\mathrm{L}_{\mathrm{I}}$, in a ; and then is a, the Reprefentation of the original Point A.

Method 2. By two vanifluing Points.
Draw $A_{1}, A_{2}$, at pleafure, cutting the Bottom of the Picture in I and 2 ; and from the Eye E, draw EL parallel to $\mathrm{A}_{1}$, and

EH parallel to $\mathrm{A}_{2}$; then draw $\mathrm{L}_{1}$ and $\mathrm{H}_{2}$, which will cut each other in a , and fo give a , for the Reprefentation of A .
Fig. 6. II. To find the Reprefentation of a Line AB , zobich is perpendicular. to the Bottoin of the Picture.

> Method i. By one vanifling Point.

Let $A B$ be the real Line upon the Ground.---No, fince $A B$ is perpendicular to the Bottom of the Picture, therefiore EC is patallel to it; and therefore C, where EC cuts the horizontal Line, is the vanifhing Point of AB.---Draw AC, and from B draw BE, cutting AC in b ; and then is Ab the Reprefentation of AB .

Method 2. By two vanifoing Pcints.
From B draw Bi at pleafure, cutting the Bottom of the Picture in I; and from the Eye E, draw EH parallel to B r cutting the horizontal Line in H ; then is H the vanifing Point of $\mathrm{Br}_{\mathrm{I}}$; therefore draw IH, cutting AC in b; which will determine the Reprefentation propofed.

In like manner, the Reprefentation Fd, of FD, which lies directly againt the Middle of the Picture, is to be determined. For C is the vanifhing Point of FD , and H is the vanifhing Point of $\mathrm{D}_{2}$.

From hence then we may conceive, that if there were ever fo many Lines parallel to $A B$, they would all vanifh into the Center of the Picture; and that the Reprefentation Fd, of any Line that fies directly againft the Middle of the Picture, will be perpendicular to the Bottom of the Picture; that is, will be Part of the Perpendicular FC, which is drawn from F to the Center C ; but in prot portion as any other perpendicular Lines (as $A B$ ) are more and more removed from the Middle $F$, the Reprefentations $A b$ of fuch Lines will be more and more oblique with the Bottom of the Picture.
III. Of a Line parallel to the Bottom of the Picture. Method I. By one vanifling Point.
Fig. 7: Let $A B$ be the Original Line.---Draw $A_{I_{3}} B_{3}$, perpendicular to the Bottom of the Picture; then is C their vanifhing Point; therefore draw $1 \mathrm{C},{ }_{3} \mathrm{C}$; and from the Extremities of the Line $A B$ draw Lines (as AE) to the Eye, cutting $1 \mathrm{C}, 3 \mathrm{C}$, in a and b ; then draw $a . b$, which will be the Reprefentation of $A B$.-Or it may be done by finding one End only of the Reprefentation (as a) and then drawing $a b$ parallel to the horizontal Line, 'till it cuts 3 C.

## METHOD 2. By two vanijbing Points.

Draw $\mathrm{A}: 2,34$, at pleafure, (but parallel to each other) cutting the Bottom of the Picture, as before: Then draw EH parallei to $\mathrm{A}_{2}$; and thenis H the wanifhing Point of $\mathrm{A}_{2}$ and $\mathrm{B}_{4}$; therefore draw $2 \mathrm{H}, 4 \mathrm{H}$, cutting IC and ${ }_{3} \mathrm{C}$ in a and b ; finally, draws the Line ab , which is the Reprefentation propofed.---Or, finding one Point only (as a) and then drawing ab parallel to the horizontal Line, a before, will be fufficient.
IV. Of a Line AB oblique with the Bottom of the Picture.

Method I: By one vanifbing Point.
Continue $A B$ to the Bottom of the Picture, and draw EH parallel thereto; and from 3 draw 3 H ; then from the Extremities $A$ and $B$ draw Lines to $E$, which will cut $3 H$ in $a$ and $b$; and then is $a b$ the Reprefentation of $A B$.

## MIthod 2. By two vaniffing Points.

From the Extremities $A B$ draw $A_{1}, B_{2}$, parallel to each other: and from E draw EL, parallel to AI, B2; then draw $1 \mathrm{~L},{ }_{2} \mathrm{~L}_{0}$ cutting 3 H in 2 and $b$; and then is $a b$ the Reprefentation of $A B$.

Here let us obferve, that fince Lines muft be either perpendicular to the Picture, parallel to the Picture, or oblique with the Picture, the three laft Examples may ferve as univerfal Rules for the Situation of all Objects that are fuppofed to lie upon the Ground; which is fully explained in the following Figure.
V. Of an equilateral Triangle, one of whofe Sides is parallel to the Pieture.
METHOD I. By baving the Original Figure ABD drawen out Fig. g. upon the Ground.
Continue BA and BD to the Bottom of the Picture, and draw EL parallel to $A B$, and EH parallel to $B D$; then draw 1 L and 4 H , cutting each other in b ; and then is b the Reprefentation of the Angle B. Again, from D draw $\mathrm{D}_{3}$ parallel to Br ; then is L its vanifhing Point; therefore draw ${ }_{3} \mathrm{~L}$, cutting 4 H in the Point $d$; and then is $d$ the Reprefentation of the Angle $D$ : And fince AD is parallel to the Bottom of the Picture, therefore, if from the Point d, the Line ad be drawn parallel to the Bottom of the Picture, it will compleat the Reprefentation propofed:---Or, it may

[^20]be done by drawing $A_{2}$ perpendicular to GL, and then drawing ${ }_{2} C$, cutting I $L$ in the Point $a$.
Method 2. By making an Angle at the Eye E, equal to the given Angle ABD..
Let ab be one Side of the Reprefentation given upon:the Pic -ture.---Continue ab to the horizontal Line; then is L its vanifhing Point: From L draw LE to the Eye; and any where a-crofs. the Line $C E$, draw ef parallel to the horizontal Line, and then make ef equal to fE : Again, draw EH through the Point e, and then is H the vanifhing Point of the Side bd ; therefore, thro: b draw bd , and from a, draw ad parallel to the horizontal Line; which will compleat the Reprefentation.---Or it may be done thus: Having continued $a b$ to its vanifhing Point $L$, and having drawn LE, make an Angle at the Eye E equal to 60 Degrees*, and draw EH, which will cut the horizontal Line in the vanifhing Point of the other Side bd.
Method 3. Witbout baving any Original Figure drawen out $f_{3}$ but by baving one Side given only.
Let ab be the Side given.---Continue ab to its vanifhing. Point $L$, and draw $L E$; then at $L$, with the Diftance LE, defcribe the Arc EH, cutting the horizontal Line in $\mathrm{H}^{\prime}$; then is H the vanifhing Point of the other Side bd; and by drawing ad parallel to the horizontal Line, the Reprefentation will be compleated.

Here let the Reader obferve again, that if the Diftance LE, of any vanifhing Point $L$, be transferred unto the horizontal Line, as LH, it will cut off one Line equal to another Line given. Thus, let $b$ a be a given Line.---From a, draw ad parallel to the horizontal Line ; and from $H$ (the Diftance of L from the Eye E) draw Hd through the Point $b$, cutting ad in $d$; then is ad equal to $a b$; for they are both the Reprefentations of two Sides $A B, A D$, of a Triangle $A B C$, whofe sides are all equal.---In like Manner, if ad was a Line given, and $L$ the vanifhing Point of a Line a $b$, which is required to be cut off equal to ad: Then make LH equal to $L E$; and from $d$, draw $d H$, cutting $a L$ in $b$; and then is $\mathrm{a} b$ equal to $\mathrm{ad} .{ }^{+}$

[^21]
## Of OBJECTG upon the Ground.

WI. Of an Equilateral Iriangle A B C, whofe Sides are all oblique Fig. 10 : with the Picture.
Method 1. By baving the Original Figure drawn out upon the Ground.
Continue the Sides of the Triangle to the Bottom of the Picture, as $\mathrm{I}, 2,3$, and draw EI parallel to $\mathrm{AC}, \mathrm{EF}$ parallel to AB , and EK parallel to BC, which will feverally cut the horizontal Line in the vanifhing Points of $A B, A C$, and $B C$; therefore, from thofe vanifhing Points draw Lines to $1,2,3$, and their mutual Interfections $\mathrm{a}, \mathrm{b}, \mathrm{c}$, with each other, will give the Reprefentation $a b c$ of the original Triangle ABC.

Method 2. By making Triangles at the Eye, as before.
Let $\mathrm{a} b$ be one Side of the Reprefentation given.---Continue it to its vanifhing Point F, and draw FE; then at E, upon the Line EF, make the equilateral Triangles MEN, MEO; continue EN and EO 'till they cut the horizontal Line, which will give the vanifhing Points required ; therefore from a, draw a I, and thre' b draw K c , which will compleat the Reprefentation a b c .
Method 3. By giving one Corner a, of the Triangle, and from tbence finding the wbole Reprefentation a bc.
From a, draw a F, and call $F$ one vanifhing Point; then from F draw F E, and at E make an Angle of 60 Degrees, and draw EI; then draw Ia, and through a, draw $f e$, parallel to the horizontal Line, at pleafure, and make af, ae, each equal to one Side of the fuppofed Reprefentation; then from the vanifhing Point $I_{\text {, }}$ fet off the Diftance IE to D; and from e, draw eD cutting aI in c ; then is ac equal to ae. Again, from $F$ fet off the Diftance $F E$, (as FP) and draw $f P$, cutting $a F$ in $b$; then is ab equal to $a f$; finally, draw $b c$, which will compleat the Reprefentation abc.
VII. Of a Geometrical Square ABCD, baving one Side A B parallel Fig. 15: to the PiEfure.
METHODI. By a Plan; that is, by baving the Original Square drawn out upon the Ground.
Draw $A C, B C$, to the vanifhing Point $C$, of the perpendicular Sides AD, BC; and from the Eye E, draw EH and EL parallel to the Diagonals BD and AC ; then from A and B draw Lines to $L$ and $H$, cutting $A C$ in $d$, and $B C$ in $c$; then draw $d c$, which compleats the Reprefentation.----Having found the Reprefentation
of one Square, any other Square, as $i k$, may be found alfo. For let ik be one Side of the Reprefentation given.---From i and k draw iC and kC ; then from i draw iL , and from k draw $\mathrm{kH}_{5}$; which will give the Depth of the Square, as in the Figure.---Or, one Diagonal only will be fufficient. Thus, AL cuts BC in c.; therefore draw cd parallel to the horizontal Line.

From hence we may obferve, that when original Squares are thus fituated, the vanifhing Points H and L of their Diagonals, are exactly as far from the Center of the Picture, as the Eye is from the Center of the Picture. Thus HC and LC are each equal to the Diftance CE ; and therefore, by fetting off CH , or CL , equal to $\mathrm{C} \mathrm{E}_{2}$ the Lines EH and EL may be omitted.
Method 2. By baving only the Depth, FI of the Square
FIL given.
Set off IL and I r, equal to the Depth FI....-.-From I and L draw Lines to $C$, and make $C L$ equal to $C E$; then draw I $L$, cutting IC in $f$; then is If cut off equal to $I_{I}$; therefore draw. $f$ e parallel to the horizontal Line $_{2}$ and the Reprefentation will be. compleated.

Method 3. By baving only one Side, as G K, given.
From G and K, draw Lines to $C$; continue $G K$, and make $\mathrm{K}_{2}$ equal to GK ; then make CH equal to $\mathrm{C} E$, and draw $\mathrm{H}_{2}$, cutting K C in b ; finally from b , draw ab parallel to the horizontal Line, and the Thing propofed is done. In like manner any other Square, mno , may be found.
VIII. Of a Geometrical Square, when its Sides are oblique with the Picture.

Fig. 12. Methodi. By a Plan ABCD.
Parallel to the Sides AB, CD, AD, BC, draw EL and EH; then are $L$ and $H$ the vanifhing Points of thofe Sides; for continue the Sides of the Square 'till they cut the Bottom of the Pictare in $1,2,3,4$; then from 1 and 2 , draw Lines to $H$, and from 3 and 4, draw Lines to L, and their mutual Interfections a, b, c, d, will give the Reprefentation propofed.-----Or the original Square may be made at the Eye, as in the Figure.

Method 2. By baving only one Side, as Gi, given upon tbe PiEfure.

Continue Gi till it cuts the horizontal Line in H , and from H draw HE; then at E, made a right Angle * with the Line HE and draw $E L$; then is H the vanifhing Point of the Sides $\mathrm{Gi}, 1 \mathrm{k}$, and $L$ is the vanifhing Point of the Sides G1 and $i k$; therefore, from G and i draw GL and i L, then from G draw GC, cutting Li in $k$; finally, from $H$ draw a Line through $k$, cutting $G L$ in 1 , and then is Gikl the Reprefentation propofed.

METHOD 3. By baving only the Length of the Diagonal eL given upon the Bottom of the Pieture, as LP.

From L draw LC; then from P draw PH, cutting LC in e; then is Le the Reprefentation of the Line LP : Again, from L draw LL, cutting PH in $f$, and from $L$ draw $L H$, then thro $0^{\circ}$ e draw $L h$, cutting $L H$ in $h$; and then chall we have the Reprefentation $L f e h$.
IX. To find the Reprefontation of a Square, a bcd, of any determinate Width; Juppofe thrce Feet.

Kig. 3.
Let a be one Corner given, and K the vanifhing Point of the Side a d.---Make KD equal to KE, and from D draw a Line thro ${ }^{\circ}$ a, cutting the Bottom of the Picture in $f$; then from $f$, on the Side of a d, fet off three Feet upon the Bottom of the Picture, and from e draw $e D$, cutting $a \mathrm{~K}$ in d ; then is a d equal to three Feet : Again, from $K$ draw $K E$, and make a right Angle at $E$, then draw EL; and then is $L$ the vanifhing Point of the Side $a b ;$ therefore draw aL, and bifect t the Angle KEL, and draw EB cutting the horizontal Line in B; then is B the vanifhing Point of the Diagonal of the Square; by which means the whole Figure may be compleated. For draw a B and d L, cutting it in c, then draw K b through the Point c ; and then is abcd the Square propofed.

[^22]X. Of a regular Hexagon *, $\begin{aligned} & \text { baving one of its Sides parallel to the } \\ & \text { Picture. }\end{aligned}$
fig. 14. Method t. By a Plan ABCDEF.
Continue the feveral oblique Sides 'till they cut the Bottom of the Picture, and draw EH and EL parallel thereto; then will L be the vanifhing Point of AB and DE , and H will be the vanifhing Point of BC and EF: Therefore, through the Corners A, D, and F, C, drawn $\mathrm{D}_{2}, \mathrm{C} 5$, which being parallel to the Sides $\mathrm{AB}, \mathrm{EF}$, will have H and L for their vanifhing Points ; and therefore, from 1 draw 1 H , and from 3 draw 3 L , cutting each other in b ; then is b the Reprefentation of the Corner B. Again, from 2 draw ${ }_{2} \mathrm{H}$, cutting 3 L in a ; then is a the Reprefentation of the Corner A , and ab is the Reprefentation of the Side A b ; therefore, draw ${ }_{4} \mathrm{H}$ and 5 L , and then is f the Reprefentation of F ; then draw a $f$, which will be the Reprefentation of AF; finally, draw 6 L , which will cut 4 H in e, and give the Reprefentation of EF ; and fo on.---Here the Learner may take Notice, that this whole Reprefentation is found in the fame Manner as the fingle Point A, Fig. 5; only the Operation in this Figure is repeated fix times, becaufe here are fix Points, which reprefent fix Corners, inftead. of one.

MeThod 2. By baving one Side ab , in the Reprejentation giver.
Through the Corner a, draw a Line $f h$, parallel to the horizontal Line, and continue $a b$ to its vanifhing Point $L$; and from L. draw LE , and make LH equal to LE; then from H draw a Line through $b$, cutting $a h$ in $h$; then is ah equal to $a b$; therefore make af equal to $a h$; and then is af the Reprefentation of the parallel Side AF: From $f$ draw $f \mathrm{~L}$, and from a, draw $\mathrm{a}_{\mathrm{p}}$, cutting $f \mathrm{~L}$ in c ; then is c another Corner; therefore draw bc , which reprefents another Side; then from $c$ draw $c d$, parallel to the horizontal Line, and draw fC cutting it in d ; then is d another Corner, and cd another Side; finally, through d draw Le, and from $f$ draw $f H$, which compleats the Reprefentation.
Fig. 15. In like Manner the Reprefentation of an Octagon (or eight-fided Figure) ABCDEFGH, is to be determined.---I have put every

[^23]Of OBJECTS upon the GROUND.

Line and Point ufed in the Operation, which, it is prefumed, is now fufficient, without any further Explanation.
XI. To find the Reprefentation of the Circle A BC.

Fig. 16;
Metho.d I. By finding the Reprefentation of Several Points, as $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathcal{E}^{2} c$.
From thefe Points draw any Lines at pleafure, but parallel to each other, as $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{~B}_{3}, \mathrm{~B}_{4}, \xi_{c}$. cutting the Bottom of the Picture ; then find their vanifhing Points, as $\mathrm{H}, \mathrm{L}$, and from each original Section at the Bottom of the Picture, draw Lines to their refpective vanifhing Points, and their feveral Interfections will give the Reprefentations of the original Points; from whence the Reprefentation of the Circle may be drawn by hand: Thus a, b, c, are the Reprefentations of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mho_{c}$. This alfo is a Repetition of Fig. 5 .
Method 2. Or the Reprefentation of a Circle may be found by means of a Geometrical Square; which is the moft ufeful Method of any.
Thus, let O be a Circle, and ABCD a Square defcribed ahout Fig. 17 it, as in the Figure.---Find the Reprefentation of that Square; which will be a fufficient Guide for drawing the Reprefentation of the Circle, to any one who has but the leaft Notion of Drawing. But if the Circle is very large, or if this Method fhould not be thought correct enough, then divide the Circle into any Number of Parts, and draw Lines through thofe Divifions parallel to the feveral Sides of the Square, as in the Figure; then by finding the Reprefentation of thofe Lines, the Appearance of the Circle may be determined with great Exactnefs.
Method 3. To find the Reprefentation of a Circle by baving only the Diameter given upon the PiEture.
Let ef be the Diameter given.----Divide ef into two equal Parts; then is $n$ the Center of that Circle. From C, and through the Points e and f , draw Cg and Ch , at pleafure, and make CL , equal to the Diftance CE; then through n draw Lg , cutting gC and hC in g and k ; finally, from the Points g and k draw two Lines, gh , ki , parallel to the horizontal Line; and then is ghik the Reprefentation of a Square equal to the Diameter of the propofed Circle; and confequently, will be a fufficient Guide for drawing its Reprefentation.

From hence then it is evident, that the Reprefentations of Circles are as eafily to be determined upon the Picture, as any other Reprefentations whatfoever; and, that after having fixed upon the Diameter of any Circle, and the Place that Circle is to poffers upon the Picture, then fuch a Reprefentation may be determined with the greateft Exactnefs, without the tedious Method of Plans, and that Infinity of occult Lines, which have hitherto been made ufe of.

What hath been faid in regard to finding the Reprefentations of Circles without a Plan, or having the original Object drawn upon the Ground, is equally applicable to any of the preceding Figures, as I have fhewn in the Courfe of this Work; and therefore, though I have put the Plans at the Bottom of each Figure, it was for no other Reafon than to explain the Truth of the Operation; and therefore the Reader will do well to exercife himfelf. with feveral Examples of the like Nature, before he proceeds to the next Section.

And here let us take Natice, that the Figures I have been putting into Perfpective, though few in Number, and the moft fimple in Nature, yet they are fuch as comprehend Forms in general.* I fay, the Forms or Shapes of Objects in general are compounded of fuch Figures as I have been reducing into Perfpective; that is, they muft be either Square, Triangular, or Circular, or elfe compounded of fome, or all of thefe put together. Thus, a Cube is compofed of fix Squares joined together at right Angles; a Pyramid, of feveral Triangles meeting in a Point; and a Column, of a Number of round Superficies laid upon each other exactly even, and perpendicular to the Ground. Thefe, and the like, nray therefore be called fimple Objects; but when they are joined together, fo as to make but one Object, then that Object may be called a Compound one: Thus a Building may be called a Compound Object; the Body of which is either a Cube or Parallelopiped; the Roof and Pediments feveral Triangles, and the Arches, Domes, Columns, $E_{c} c$ are nothing elfe but Circles, or Parts of Circles, put together. And thiercfore, it follows from hence, that whoever is able to put a Square, a Triangle, or a Circle, rightly into Perfpective, has got all the Materials that are neceffary for drawing the Reprefentation

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of any Object upon the Picture. I fhall therefore follow this fimple Method throughout this Work, and will now proceed to Thew, how to determine the Reprefentation of Objects when they ftand perpendicular to the Ground; which is the Subject of thenext Section.

## S E C T. IIF.

Of OBBIECTs that are perpendicular to the GROUND:

1. To find the Reprefentation of Planes when their Bafes are perpen- Fig. isis dicular to the Bottom of the Pisture, like AB, Fig. 6. .
Case 1. When one Corner A ; of the Plane ABCD , is at the Bottom of tbe Piczure :---And let it be required to drawo the Reprefentation of a Plane fix Feet bigh and four Feet wide.

FRom A, upon the Bottom of the Picture, make a Scale of Feet*, at pleafure, as in the Figure; and from the Point A draw $A D$, perpendicular to the Bottom of the Picture, and continue it at pleafure ; then upon the Line $A D$ fet fix Feet from $A$, and draw DC to the Center C, and from A draw AC; then make CH equal to CE , and draw $\mathrm{H}_{4}$ cutting AC in B ; and then is A B equal to four Feet; therefore, from B draw B C, parallel to. $A D$; and fo will $A B C D$ reprefent a Plane four Feet wide and fix Feet high.
CASE 2. Let it be required to drawe the Reprefentation of a PlaneGHIK, three Feet Square; and let the neareft Corner G, be one Foot from the Bottom of the PiEture.
Draw any Line at pleafure, as C 9 , and make CL equal to CE; then fet off one Foot from 9 to 8, and draw L 8 cutting C 9 in G ; then is G one Foot from the Bottom. Again, take three Feet more, as 85 , and draw 5 L cutting GC in K ; then is GK equal to three Feet ; therefore, draw G H and IK perpendicular to the Bottom of the Picture, and continue them at pleafure; and from G. draw GF, parallel to the Bottom of the Picture; then is GF equal to three Feet; finally, make GH equal to GF, and draw HC ; which will compleat the Reprefentation propofed..--Or it may be done thus : From 9, draw 9 N perpendicular to the Bottom of

[^25]the Picture, and make it equal to three Feet; then draw 9 C and NC, and from 8 draw 8 L , which will give the Point, or Corner, G ; then draw 5 L , which will give three Feet for GK ; therefore, by drawing GH and IK parallel to 9 N , the Thing propofed is done.--In like Manner, fuppofe a Plane abcd, four Feet fquare, was removed five Feet into the Picture, and fuppofe the lower Edge, or Plan, to be fomewhere in the Line AC.--From A, where it cuts the Bottom of the Picture, fest off five Feet for its Diftance, (as A 5) and four Feet for its Width (as 59 ; ) then draw $\mathrm{H}_{5}$. and H 9 , cutting AC in a and b ; then is ab the Reprefentation of its Depth; therefore, draw AD, ad, bc, perpendicular to the Bottom of the Picture, and make AO equal to four Feet; and then draw OC, cutting ad and $c b$, which will compleat the propofed Reprefentation.
II. To find the Reprefentations of Planes that are parallel. to the Picture.

Eig. 19. CAsE 1. For a Plane ABDE, four Feet fquare, which, we fuppofe, is removed two Feet from the Bottom of the Picture.
Divide the Bottom of the Picture into any Number of Parts, which call fo many Feet ; then from G and 4, draw GC and 4C, and make CL equal to CE, and draw 6 L cutting 4 C in E ; then is 4 E equal to 46 , that is, equal to two Feet; therefore, draw EA parallel to the Bottom of the Picture, which will cut GC in A, and give the Length of one Side, upon which make the Square ABDE, which will be the Reprefentation propofed.

Case 2. For a Plane fix Feet bigh and three Feet wide, wofich is to be five Feet from the Bottom of the Picture.
Any where, at pleafure, draw $\mathrm{C}_{4}$, and fet off three Feet, (as 47) then draw 7 C , and from 4 fet off five Feet, (as 49) and draw 9 L , which will cut ${ }_{4} \mathrm{C}$ in e ; then will e 4 be equal to 49 , that is, to five Feet. Again, from e, draw ef parallel to the horizontal Line, which will give ef equal to three Feet; that is, equal to 47 ; therefore, continue ef at pleafure, and call a, one Corner of the intended Plane; make ab equal to ef, and draw ad, bc, perpendicular thereto; then make ad equal to twice ab, and draw dc parallel to the horizontal Line, and then will the Reprefentation be compleated.
III. To find the Reprefentation of Planes, when their Plans or Bafes are oblique with the Bottom of the Picture, like AB, Fig. 8.
Let ABDF be the Reprefentation fought, which let be fix Feet Fig. 20: high, four Feet and a Half wide, and one Foot and a Half from the Bottom of the Picture ; and let L be its vanifhing Point, and LG the Line in which the Plane is to ftand.---Draw G I perpendicular to the Bottom of the Picture, and fet fix Feet upon it (as in the Figure) and draw IL; then from $G$ fet off Ga equal to one Foot and a Half, and draw aH cutting GL in $A$; then is GA equal to Ga ; that is, equal to one Foot and a Half. Again, from a, fet off four Feet and a Half, (as a 6 ) and draw 6 H cutting GL in $B$; then is $A B$ equal to a 6 , that is, equal to four Feet and a Half; therefore by drawing AF and BD perpendicular to the Bottom of the Picture, we fhall have the Reprefentation of a Plane, fix Feet high, and four Feet and a Half wide.

If the vanifhing Point $L$ is out of the Picture, the Figure may Fig. 21, be draw thus.---Let BA be the Reprefentation of one Side given, and $A D$ its Height.---Continue $A B$ at pleafure, and any where upon it draw ab perpendicular to the Bottom of the Picture ; then make cb to ca , as FD is to FA , and draw Db ; which will give the Length of BC; for if DC be continued it will vanifh into $\mathrm{L}^{*}$.

From hence then it is evident, that the Reprefentation of any perpendicular Plane may be immediately determined upon the Picture, without having Recourfe to the tedious Methods of Plans, Elevations, $\mathcal{E}_{6}$. and but very few Lines are required, even when the Reprefentation is to be of any given Dimenfion, or however it is to be fituated upon the Picture: But, if the Reprefentation is not to be of any particular Dimenfion, being left to the Difcretion of the Artift, then nothing can be more fimple than the Operation. For let $A B$ be one Side given, and $A D$ its Height; then from A Fig. 18, and B draw $\mathrm{AD}, \mathrm{BC}$, perpendicular to the Bottom of the Picture; and from C, draw CD; which will compleat the Figure.

Here let us obferve again, that the vanifhing Point $C$ of the Line $A B$, is the vanifhing Point of every Line DC, $N M, \mathcal{E} c$. that is parallel to $A B$; agreeable to the fecond Axiom.

Having fhewn how to find the Reprefentation of Square Plancs perpendicular to the Ground; let us now proceed to join them together, which begins the Perfpective of folid Figures.

[^26]IV. To find the Reprefentation of Triangular Pieces of Wood, ofc. roben they are citber above or below the Horizontal Line.
Fig. 22. CASE I. When they are below the Horizontal Line, and bave one Side, abed, parallel to the Picture.
Find the vanifhing Points H and L , of the oblique Sides, as taught in Figure 9; then from b and d , draw $\mathrm{bL}, \mathrm{dH}$, cutting each other in c .; then is bcd the Top; which compleats the Figurc. In this Figure the Front Side abde, is parallel to the Picture, but in the Figure nopqrm, the back Side npmq is parallel to the Picture ; therefore let us find the Appearance of that alfo:-Here let $m q p n$ be the parallel Side given.-.-Through the Points $n$ and $m$ draw Lines from $H$, and through the Points $p$ and $q$ draw Lines from $L$, which will cut the Lines $\mathrm{Hm}, \mathrm{Hn}$, continued in o and $r$; therefore draw or, and the Thing propofed is done.Or it may be found by giving one Edge $m \mathrm{r}$.---Continue mr to its vanifhing Point H , and draw rL ; then draw mq parallel to the horizontal Line, cutting $r \mathrm{~L}$ in q ; then is rq the other Side; therefore draw $\mathrm{mn}, \mathbf{r e}, \mathrm{pq}$, each perpendicular to the horizontal Line, and make any of them the propofed Height, (fuppofe ro) then draw $\circ \mathrm{H}$, oL, cutting mn , and qp , in $n$ and p , and then drew np; which fnifhes the Figure.

## CASE 2. When they are above the Horizontal Line.

Let fg be one of the Bottom Edges given.--Draw fi perpendicular to the horizontal Line, and make it equal to the propofed Height; from $f$ draw $f L$, then draw $g l$ parallel to the horizontal Line, cutting $f \mathrm{~L}$ in 1 ; and then is fl the Depth of the other Side ; therefore, draw gh and lk parallel to fi , and from i draw iH and iL ; which compleats the Reprefentation.
V. To find the Reprefentation of any triangular Figure, when all its Sides are oblique with the Piature.

Fig. 23.
Case I. When it is below the Horizontal Line.
Let AD be one Edge given, whofe vanifhing Point is out of the Picture, and let L be the vanifhing Point of the other Edge A G.---Make AG equal to AD, by Figure 10; and from $A, D, G$, draw Lines perpendicular to the Ground; then make $A B$ equal to the Height, and draw BL; which will compleat the Side ABFG: Again, continue $A B$ and $D E$ to the horizontal Line, and make DE
$D E$ to $D H^{*}$, as $A B$ is to $A R$; then draw $B E$, which compleats the Side A BiED; finally, draw EF, which finifhes the Reprefentation. For, if $A D$ and BE were continued, they would both vanifh into the fame Point in the horizontal Line ; as was obferved. in Fig. 21.

> CASE 2. When it is above the Horizontal Line.

Let HO be one Edge given, whofe vanifhing Point is out of the Picture, and let L be the vanifhing Point of the Edge HN.... Make HN equal to HO , as before ; and from $\mathrm{N}, \mathrm{H}, \mathrm{O}$, draw. Lines perpendicular to the Ground, then make FK equal to the propofed Height, and draw KL, which will give one Side : Again, make PQ (the Part above the horizontal Line) to PO (the Part below the horizontal Line) as IK is to IH (which in this Figure is as 2 is to 3 ). and draw $\mathrm{K} \mathrm{Q}_{\text {; }}$; which will compleat the Re prefentation propofed. For if HO and K. Q were continued, they would meet in a Point upon the horizontal, Line..

This Method of determining the Appearance of any Line, when its vanifhing Point is out of the Picture, is extremely ufeful ; and therefore, the Reader cannot make it too familiar to him; the general Method for which I have farther explained in Figure 4 th
VI. To find tbe Reprefentations of Cubes, both above and below the Horizontal Line, when fome of their Sides are parallel to the PiEture.

## Case 1. When below the Horizontal Line.

Let abeg be one Side given.----Draw b C, e C, and gC, and Fig. $2_{4}$ : find the vaniming Point $H$ of the Diagonal ef, by Figure II; and draw eH cutting Cg in f ; then from f , draw fd parallel to the horizontal Line, cutting $\mathrm{e} C$ in d ; and then draw d c parallel to $\mathrm{e} b$, and the Reprefentation is compleated. And for the Cube E--Make the front-fide like the Cube A, and draw Lines from the upper Corners to the Center $C$; then by continuing eg and $f d$ we may compleat the other alfo; as in the Figure. And therefore, having got one Reprefentation, That will be fufficient for any, Number of the fameKind, provided they ftand all in the fame Line, 2 n ; that is, at the fame Diftance from the Bottom of the Picture.

[^27]C As e 2. When abave the Horizontal Line, as B and D.
Here let the Reader obferve, that the Rule in either Caife is the fame; and therefore he is to proceed in the fame Manner in finding the Reprefentation of a Cube above the Eye, as we have done: in determining the Appearance of a Cube below the Eye; which is fufficiently explained by the Figures:--And fo likewife for the Depth mo , of the Parallelopiped F ; which is found by drawing a Line from the Corner $n$, to the vanifhing Point of the Diagonal H .
VII. Of a Cube and Parallelopiped, whofe Sides are all obligue with the Picture.

## CASE I. Of the Cube.

Fig. 25. Let a b be given, whofe vanifhing Point is L ; and let $H$ be the vanifhing Point of the other Side ag.---From a, draw a 1 parallel ito the horizontal Line, and make LA equal to the Diffance LE; then through $b$, draw $A I_{\text {, cutting } A} A$ in $I$; and then is $a \cdot x$ equal to $a b$; and from a and $b$ draw Lines perpendicular to the horizontal Line, and make a e equal to a. 1 ; then from e, draw e $L$, cutting $b \cdot \mathrm{in} c_{5}$, and then we hall have one Side: Again, from $c$ and e, draw Lines to $H$, and from e, draw a Line to $C$, (the vanifhing Point of the Diagonal) which will cut cH in d; then from $L$ draw a Line through $d$, cutting e $H$ in $f$; finally, from $f$, diaw $\{g$ parallel to $a e$; and then will the Reprefentation be compleated.

CAss 2. Of a Paralletopiped, or oblong Piece of Wood, refting upon one of its longeft Faces:
Let it be required to make it three Feet long, one Foot thick, and one Foot high: And let o be the neareft Corner, and $\mathrm{H}, \mathrm{L}$, the vanifhing Points of the Sides.--Through o, draw 203 , parallel to the horizontal Line, and fet off three Feet upon it; then draw $\circ \mathrm{H}$ and oL , and make HB equal to HE, and draw B2 cutting: ${ }^{\circ} \mathrm{H}$ in i ; then is oi equal to three Feet: Again, make A equal to LE, and from 0 , fet off o3, equal to one Foot; then draw 3 A, cutting oL in $n$; and then is on equal to o 3: Again, from i, $o, n$, draw Lines perpendicular to the horizontal Line, and make om equal to o 3 ; finally, from $m$, draw $m H$ and $m L$, cutting ih in $h$, and $n 1$ in 1 ; then from 1 draw a line to $H$, and from $h$ draw a Line to $L$, which will cut each other in $k$, and fo finifh the Reprefentation; which will be three Feet long; one Eoot thick, and one Foot high.

WIIL TO

QIII. To find the Reprefentation of, an Hexangular Figure, both above and belowe the Eye:
Case r. When below the Eye.
Let ab be one Side given, and let H and L be the vanifhing Fig. 26 . Points of the other Sides.--Continue $a b$ on cither Side, at pleafare, and make a.r, b.2, equal to $a b$; then cut off $a . e$ and $b f_{2}$. equal to La a and 2 b ; and from.e, $a, b, f$, draw Perpendiculars to ab ; then make: ad equal to the propofed Height, and draw cd . parallel to $a b$; then from $c$ and $d$, draw Lines to $L$ and $H$, cutting $f h$ and $e g$ in $h$ and. $g$; and from: $g$ and $h$, draw Lines to: $L$ and. $H$, and from $c$ and $d$, draw Lines to $C$, cutting them in $i_{2}$ and k ; finally, draw ik , which will be parallel. to cd , and will be the Reprefentation propofed.

## Case 2. When above the Eye.

Let mn: be one Side given, and $H, L$, the vanifhing Points of: the other Sides, as before.--Continue mn , at pleafure, as 3.4 , from: whence the ather Sides may be found, and confequently, the whole Reprefentation; as is evident by the Figure:: In which I. have put every Line in the Operation, to make it eafy to be underftood without any further Explanation.

From hence then it follows, , that the Appearance of any Objects. may be as eafily determined above the horizontal Line as below it; fince one Rule ferves in both Cafes; and therefore it matters not. whether we begin our Work at the Bottom or at the Top of the Picture. Now, this Method of finding the Reprefentation of $\mathrm{Ob}-$ jects is of prodigious Ufe.. For fuppofe it was required to draw. the Reprefentation of the Top of any Building; we need not fketch out any more of it than is to appear upon the Picture; but we may: begin in the very Place where that Top is to be, without undergoing the tedious Tafk of beginning at the Bottom of fuch at Building, and afterwards rubbing out what is not to appear.

## IX. To put an Octangular Building into. Perspective.

Let ab be one Side given, and let $\mathrm{H}, \mathrm{C}, \mathrm{L}$, be the vanißhing Fig. 77. Points of the feveral Sides.---Find the Reprefentations of the Sides, as ah , bc, and cd , which are vifible to the Eye (by Fig. 15.) and from the feveral Points $h, a, b, c, d$, draw Perpendiculars to the Bottom of the Picture; then make ak equal to the propofed Height, and draw kg parallel to ab , which compleats one Side; then from $k$
draw kH , and from g draw gL , which finifhes two Sides mores finally, from $f$ draw $f C$, which will compleat the whole Reprefentation.
X. To find the Reprefentation of Cylindrical, or round Objeets, fuch
as Columns, and the like.

In the 16 th and 17 th Figures we have fhewn feveral Methods of funding the Appearance of Circles upon the Picture, by which means the Reprefentations of Circles of any Dimenfions may be determined with great Exactnefs; and fince a Circle is the Bafe of a cylindrical Object, therefore, by finding the Reprefentation of two Circles at any determinate Diftance, the Appearance of that Object may be determined alfo.
Fig. 28. Let it be required to find the Reprefentation of a round Object like D.

## CASE 1. When it fands upon one End.

Let ef be the Diameter given, and $n$ the Center of the Circle. ---Draw nb perpendicular to the Bottom of the Picture, and make it equal to the Height you intend for the Reprefentation; then, by Method 3, Fig. 17, find the Reprefentation of the Square ghfkie ; which will be a fufficient Guide for drawing the Appearance of the Circle, as in the Figure. Again, through b, draw cd parallel to ef, and, by the fame Method, find the Reprefentation of the Square al, which will be a Guide for the upper Circle; finally, from the Extremities of both Circles, draw ec, fd, parallel to $n b$, which will exhibit the vifible Appearance of the round Object, as in Figure D.

From hence then it is manifeft, that any Number of round Objects, (fuch as Columns, $\mathcal{E}^{\circ}$ c.) may be found upon the Picture; by having only their Diameters and perpendicular Heights, as we have further fhewn in Figure 63 , E8c.
Ekg. 29. CASE 2. When a Cylinder lies upon the Ground oblique with the
This alfo may be done with the greateft Eare, by finding the Appearance of two geometrical Squares. Thus, let A be the Corner of the Square for the neareft End of the Cylinder, $A B$ its Diameter, and AC its given Length; and let H be the vanifhing Point of the End, and $L$ the vanifhing Point of the Sides.---Cut off $A E$ equal to $A B$, and from, the Points $A E$ draw Perpendisulars, and compleat the Square, a; then draw its Diagonals, Ec.


25.

27.

and then the Appearance of the Circle, as in the Figure. And for the Length, cut off $A D$ equal to $A C$, and from the Point $D$. make another Square for the fartheft End, in which draw the Appearance of another Circle; then draw Lines from the Extremities of the Circle a, to L, which will eut the Circle in the Square at $D$, and thereby compleat the Reprefentation as required.

If there hould not be Room enough for the whole Draught below the horizontal Line, it may be done above it, as in the Figure, taking great Care to make them both at the fame Diftance from it.-2Thus, $\mathrm{c} n$ is equal to cA .

From hence it appears, that this Figure is determined after the fame. Manner as the oblong Piece of Wood, Fig, 25; only, one is fquare and the other circular.

Thefe Examples are fufficient to thew the whole Practice of Perfpective, to far as it relates to Objects which lie flat upon the Ground, or are perpendicular to it : For, as I obferved before, the immediate Objects of Perfpective, are a Triangle, a Square, and a Circle ; and therefore were we to multiply Objects to infinity, they would be compounded of fome or all thefe put together ; and confequently, what has been faid already, is fufficient for our Purpofe.

## SECT. IV.

Of ObJects which are inclined to the Ground. fuch as Pediments, Roofs of Houfes, and the like.

TTHIS Part of Perfpective, neither the Fefuit, nor Pozzo, not many others, feem to haye had the leaft Knowledge of; for they have confined themfelves wholly to the horizontal Line, without confidering any other vanifhing line; and therefore, when they have fhewn how to find the Appearances of inclined Objects, they did it by means of Plans, Elevations, Gec. which is not only a tedious, but an uncertain Method. But, Dr. Taytor has fhewn us, that inclined Objects have their proper vanifhing Lines and Points, as well as thofe Objects which fie flat upon the Ground, or are perpendicular to it ; and that the Method for determining the Appearance of Objects in either Cafe, is exactly the fame. Which we are now going to demonftrate.

Fig. 30. I. To find the Reprefentation of a Square, by means of its Diagonals onl;, woben it is jituated like G HIK, Fig. 18:

> METHODI. Let ab be the lowaer Edge.

Continue $a b$ to its vanifhing Point $C$ and through $C$, draw EV: perpendicular to HL, which will be the vanifhing Line of the propofed Plane; then make C© equal to the Diftance of the Eye, and draw AB parallel to the horizontal Line, upon which make the Square $A B C D$ of any convenient Bigness, and draw its Diasgonals $A C, B D$; then from $\mathbb{E}$, draw $\mathbb{E} E$ and $\mathbb{E}$ parallel to the Diagonals $\mathrm{AC}_{2} \mathrm{BD}$; and where they cut the vanifhing Line $\mathrm{EV} \mathrm{V}_{2}$ will be the vanifhing Points fought: Thus, E is the vanifhing: Point of the Diagonal ac, and $V$ is the vanifhing Point of the Diagonal bd.

## MeтноD 2. By making a giaen Angle at the Eyf:

The Angle of a Square is a right Angle, and contains oo De, grees, the half of which is 45 Degrees; therefore, at $\mathbb{E}$, with the horizontal Line make two Angles, C巴E and C©V, each equal to 45 Degrees, then draw $\mathbb{H} \mathrm{E}, \mathbb{V}$, cutting the vanifhing Line in $E$ and $V_{i}$ which will be the vanighing Points, as before.

METHod 3. By the Difance of the Eye only.
Through the vanifhing Point $C$, draw the vanifhing Line EV, and make CE, CV, each equal to the Diftance CE of the Eye 3 which gives the vaniming Points propofed.

From hence then it appears, that Planes which are perpendicular to the Picture, and to the Ground alfo, will have their vanifhing Lines pals through the Center of the Picture, perpendicular to the horizontal Line; and that all the oblique Lines which can be drawn within thofe Squares, will vanifh into this Line, for the fame Reafon that all the oblique Lines which can be drawn within 4. Square that lies upon the Ground, will vanifh into the horizontal Line. And from hence alfo we may conceive, why Roofs, Pedion ments, Eoc, will have their proper vanifhing Points as well as any, other Objects. For let a be be the End of a Roof or Pediment, then is $E$ the vaniming Point of the Side a $e$ which is next the Eye, and $V$ is the vanihing Point of the other Side $b e$; and if Lines ara drawn through $E$ and $V$, parallel to the horizontal Line, then thefe Lines will be the vanifhing Lines of the Sides of the Roof, for the fame Reafon that EV is the vanifhing Line of its Ends: As is evident from the next Figure.
11. To find the vanijfing Lines and vanifhing Points of a Roof, wiben Fig. 3r: the Eind of the Building is Jituated like abcd in the laf Figure,
Draw the vanifhing Lime IJ, as before taught, and make C巴 equal to the Diftance of the Picture---Paratlel to the horizontal Line, (or if you pleafe upon the horizontal Line) draw $A B$, upon which, draw the End of the Roof, as $A B C$; then from $\mathbb{E}$, draw $\mathbb{E} V ; E$; parallel to $A C, B C$, cutting the vanifhing Line in $V$ and if then are $V$ and L the yaniming Points of the inclined Edges acc, bc. Again, through Y and $L$ draw aix and 1 parallel to the horizontal Line, and then will uid be the vanifhing Line of the inclined Side aecd, and vl will be the yanifhing Line of the inclined Side bcd.

Now in order to find any vanifhing Point upon either of the vanifhing Lines iale, or vl, we muft proceed exactly in the fame manner as in finding any vanifhing Point upon the horizontal Line ; namely, by fetting off the Diftance of the vanifhing Line, and then drawing Lines from thence parallel to any original Eines whofe vaniming Points are required. Thus, let it be required to find the vanifhing Points of the Diagonals of a Square, whofe: Sides vanifh into the Center V, of the vaniming Line ©ial ; like $a b, d c$, Figure $32,--M a k e V Y$ equal to. VE, and LJ equal to Lé then at ${ }^{-1}$ and $\mathrm{J}_{3}+$ with the Lines VI , LJ , make Angles of $45^{\text {. De- }}$ grees each, as in the Figure, and draw the Lines IOI, II, Jv and: J1'; which will give the vanifhing Points propofed.--Or it may be done by making, $\mathbb{G} V$, ${ }_{3} V$, E゚c. equal to the Diftance $V \mathbb{E}$, which comes to the fame Thing. For fuppofe the Picture removed into the) Place of the 3ed Figure; where uly is the vanifhing Line of Fig. 3n the Square abfg , and and ou the vanifhing Points of its Diagonals; and let ab be one Edge of the Square, which fands upon the Groundo-Froma and $b$ draw Lines to $V$, which is the vaniffing Point of the Sides $a e ; b d$; and then from $b$ draw $a$ Line to $d x$ and from $a^{\prime}$, draw a-Line to llating $a V$ and $b V$ in $f$ and $g$ : finally, draw $\dot{g} f$; and then will abfg be the Reprefentation of a Square inclined to the Ground, like the Line AC, Fig. 31. And in like Manner, if another Square was required, as fged, it may be found by repeating the laft Operation; that is, by means of the Diagonals, as is evident by infpecting the Figure: Or any Number of Squares may be found by the fame Method: From whence-it is manifeft, that the Reprefentation of any inclined Object may: wery eafily be determined, and made of any given Proportion.

And what has been faid about the inclined Side abed, is equally: applicable to the oppofite inclined Side; fince the only Difference confifts in working below the horizontal Line, inftead of above it : For $v \mathrm{v}$ is its vanifhing Line, and $v$ and 1 the vanifhing Points of the Diagonals, Ec.--I have added the Figure A, which reprefents, as it were, the Frame-work of the other; and will ferve to explain the Thing more fully.

The principal Difficulty in determining the Reprefentation of any inclined Planes, lies in finding the Center and Diftance of their peculiar vanifhing Lines; therefore, before we proceed any further; we will give fome general Rules for that Purpofe, as is moreover explained by the 50th Figure.

## 1. To. find the Center of a vanifbing Line.

Fig. 34.
Let © $L$ be avanifhing. Line given.--From $C$ the Center of the: Picture, draw CH perpendicular to the vanifhing Line $\mathfrak{L A}$, ands then is H the Center of that vanifhing Line. Again, let dal be a vanifing Line given.---From C the Center of the Picture, draw CO perpendicular to aiz, and then is $O$ the Center of that vanifh ing Line.

## 2. To find the Difiance of a vanibing Line:

Continue the Perpendicular CH , at pleafure, towards $\mathbb{E}$; and from the Center of the vanifhing Line draw HE to the: Eye; then is HE the Diftance of the vaniPhing Line La ; therefore, fet off $H \mathbb{C}$ equal to the Diftance HE, and then is $\mathbb{E}$ the Diftance to be work'd with. Again, for the vanifhing Line al Perpendicular CO towards I, at pleafure, and from O fet off OJ upon the vanifhing Line, equal to $C E$, the real Diftance of the: Eye, and draw CJ; then is CJ the Diftance of the vanifhing Line: EA I ; and by making OI equal to CJ, we fhall have I for the Point of Diftance of the vanifhing Line al w.. From hence then it will appear, that $C$ is the Center of the horizontal Eine in the 30th Figure, and it is alfo the Center of the vanifhing Line EV; that CE is the Diftance of the Picture, (that is, of the horizontal. Line) and $C \mathbb{E}$ is the Diftance of the vanifhing Line EV: And becaufe the vanifhing Line EV paffes through the Center of the Picture, therefore the Diftances CE and Ce muft be equal. Again, $V$ and $L$, of the 3 rft Figure, are the Centers of the vanifhing Lines $3 i$ il and V1; and VI, L J, are their Diftances, and fo on. Thus much is fufficient for our prefent Purpofe; but in the-
soth Figure I have given one general Rule, not only for determining the Center and Diftance of each vanifhing Line, but for finding the vanifhing tine of any Plane, let its Inclination be what it will : All which thould be well remembered.

Having determined the Center and Diftance of any vanifhing Line, we are then to proceed with our Work in the very fame Manner as in drawing the Reprefentations of Planes that lie flat upon the Ground ; and, by turning the Figures, we may conceive every vanifhing Line to be a horizontal Line, © $\mathrm{O}_{\text {c }}$.
III. To find the Reprefentation of a Square a bed, by the vanifbing Fig. 33 : Points of its Diagonals, when it fands perpendicular to the Ground but oblique with the Picfure, like A B DF, Fig. 20.

## METHOD i. By drawing out a Square.

Let ab , be the under Side given.--Continue ab to its vanihing Point O , and through O , draw $I \mathrm{~J}$ perpendicular to the horizontal Line; then is IJ the vanifhing Line of the Square. Again, from E draw EO, which is the Diftance of the vanifhing Line IJ; therefore, fet off $O \mathbb{E}$ equal to $O E$, and parallel to the horizontal Line draw a Line, as AB , at pleafure; upon which, make a Square $A B C D$, and draw its Diagonals; then from © draw Lines parallel to thofe Diagonals, which will cut the vanifhing Line in I and $J$; and then are I and J the vanihing Points of the Diagonals $a c$ and $b d$.

## Metatod 2.

Make at $\mathbb{\mathbb { E }}$, on each Side of the horizontal Line, an Angle of 45. Degrees, and draw $\mathbb{E} \mathbb{I} \mathbb{E} J$, which will produce the vanihhing Points propofed.

## Method $3^{-}$

Or, the vanifhing Points may be determined in this Example by making OI and OJ equal to O ©.
IV. Suppofe a be to reprefent the End of a Roof, as hefore; then the Sides of that Roof weill be obligue with the Eye, like a bde, Fig. 34; therefore, let us next find the Reprrefentation of a Plane fituated in this.Manner.
Method i. Let ae be the lower Edge of the Roof, which let us mig. 34 Juppofe to be a Square that refts upon the Edge ae. Continue the Side ae to its vanifhing Point I , and draw $\mathrm{L} \mathrm{C}_{\mathrm{t}}$; then at E make a aight Angle, and draw EH; and then is $\frac{1}{2}$ the Book II.

## Of OBJEGTs inclined to the GBoun Pe

vanifhing Point of one Side of a Square which lies upon the Ground，and $H$ the vanihing Point of another Side of the fame： Square；therefore，from the Corner a draw $\mathrm{aH}_{2}$ and through H draw en perpendicular to the horizontal Line；then is H the Center of the vanifhing Line $\mathrm{ULL}_{2}$ ，and HE its Diftance：Again， make HE equal to EH，and make AC parallel to it then upon AC draw the End of the Roof，（that is，the Angles of its Incli－ nation）and from $\mathbb{C}$ draw $\mathbb{C} \mathbb{U l}$ ， $\mathbb{E} L$ ，parallel to $A B, C B$ ，which will cut the vanifhing Line $\mathfrak{a l}_{L_{2}}$ in the vanifhing Points oll and $\mathrm{L}_{5}$ ． and fo will $\mathfrak{a l}$ be the vanifhing Point of the Edges ab，ed，and I will be：the vanifhing Point of the Edges ae，bd，and L will be the vanifhing Point of the Edge cb ：And if a Line be drawn from da to IL，it will be the vanifhing Line of the inclined Face a ebd； from whence it is evident，that after we have found two vanifh－ ing Points of any inclined Plane，if a Line be drawn through thofe Points，it will be the vanifhing Line of that Plane．But to com－ pleat the Reprefentation propofed：Find the Center of the vanifh ing Line $\boldsymbol{\sigma} \boldsymbol{l} \boldsymbol{l}$ ，and fet off its Diftance upon the Perpendicular OI； then draw Lines from the vanifhing Points $\mathfrak{O}$ and 亚，which will make a right Angle at the Eye；then bifect that Angle by the Line： ID，which will give $D$ for the vanifhing Point of the Diagonal of a Square ；by which means the Square aebd may be compleated．

Method 2．By baving the Width，as ac，of the Roof given．
Let ac be the Width，or what is generally called the Span，of the Roof；and let and $L$ be the vanifling Points of the Roof． －－－From a，draw a $\mathfrak{l l}$ ，and through c draw $L$ b，cutting a ili in b ；then is abc the Reprefentation of one End．Again，from H draw HE，with which，at E，make a right Angle，and draw E 4 ； then is 3 业 the vanifhing Point of the Side ae which refts upon the Ground；therefore，draw all and bil then find the vanifhing． Point of the Diagonal of a Square，whofe vanifhing Points are H and 欮，and from a，draw aD，cutting bil in d；then from draw a Line through $d$ ，which will cut alt in $e$ ，and thereby com－ pleat the Reprefentation propofed．
Method 3．By cutting off one Line equal to another Line given．
From the Corner a draw af parallel to the vanifhing Line atil，and make af equal to one Side of the intended Square； then fet off $\mathfrak{X i} F$ equal to $\mathfrak{X I}$ ，and draw $F f$ ，cutting $a \mathfrak{a l}$ in $b$ ；and then is $a b$ equal to $a f$ ．Again，from $a$ and $b$ draw
araw all; by, and from a draw ag parallel to the horizontal Line, and make it equal to $a f$; then fet off $\mathcal{G} G$ equal to ${ }^{2} E$ and draw $G g$, cutting amin $e_{j}$; then is ae equal to a $g$, that is, equal to af; therefore, draw e d, which compleats the Square abed; finally, draw aH and bL, which will finigh the whole Figure.
Metaod 4. By baving the vanifhing Line ua given, at pleafure.
From the Center of the Picture draw CO perpendicular to the vanifhing Line $\mathfrak{U}$, and fet off the Diftance of the vanifhing Line fiom $O$ to $I$, and let $a b$ be one side given,--Continue abstill it cuts the vanifhing Line in its proper vanifhing Point $\mathbb{U}$, and from ditraw II, then at I make a right Angle, and draw I. 3 ; and then is It the vanifhing Point of the Sides ae, bd; and by finding the Point $D$, the Square may be compleated, as before. Again, for the upright End;-Continue the horizontal Line at pleafure, and make © equal to 1 I, cutting the horizontal Line continued in $\mathbb{E}$; then is $\mathbb{E} H$ the Diftance of the vanifhing Line UL $L$ by which means the vanifing Point $L$ of the Side $b c$ may be determined:--Or the vanifhing Points of any Lines may be found upon Ul 理, by infcribing a Figure at the Eye, like the Original of our propofed Reprefentation; as the Square I. Now what is faid of a Square, will ferve for any other Figure; which, it is prefumed, is now fo evident as to need no farther Explanation; efpecially, fince a little Practice will make all that has been advanced in this Section very eafy and familiar.

Here let us obferve, that when Objects are parallel to the Ground, they will have their feveral vanifing Points in the horizontal Line; when they are perpendicular to the Ground, they will vanifh into a Line perpendicular to the horizontal Line, like Fig. 30, $31,32,33$ : when they are inclined to the Ground, but have fome of their Edges parallel to the Picture, like a $b$, ed, Fig. 32. they will then vanif into Lines parallel to the horizontal Line; and will be above the horizontal Line when the Plane leans from the Eye, and below the horizontal Line when the Plane leans towards the Eye, but when the inclined Planes are every way oblique with the Picture, the Eye, and the Ground, like Fig. 34, then the vanifhing Points of their feveral inclined Sides will vanifh into Lines aflant the horizontal Line, like da IL Now, thefe being all the Variety of vanifhing Lines which can ever happen in common Practice, it were needlefs to produce any other Examples
of this Kind : But to affirt the Curious in determining the Reprefentations of Regular Solids, * or fuch-like complicated Bodies, I have added the fix following Figures; which may be omitted by the Generality of my Readers, as Things more curious than ufeful', and which are not in the leaft effential to common Practice; and. therefore, they are now referred to the next Chapter.
Fig. 35. V. Te find the Reprefentation of a Cube that refts upon one of its Edges a b.
EXAMPLE I. When fome of its Edges, as $a b, c d ; f e$, are parallel? to the PiEture.
Let ab be one Edge given, which let us fuppofe refts upon the Ground. Now, becaufe the Edges $\mathrm{ab}, \mathcal{E} c$. are parallelito the Picture, therefore the End adf $g$ will be perpendicular to the Picture; and confequently, the vanifhing Line VL of that. End will pafs. through the Center of the Picture, and will be-perpendicular to the horizontal Line: And if we fuppore the Diagonal af to be perpendicular to the Ground, then the vanifhing Point of the other Diagonal dg will be the Center of the Picture, becaufe it is parallel to the Ground. Therefore, through C draw the vanifhing: Line VL, and make C $\mathbb{E}$ equal to the Diftance of the Eye; then at $\mathbb{C}$ make a Square, in fuch a manner that its Diagonal I 2 may be parallel to the vanifhing Line VL; or, which is the fame thing; make the other Diagonal a Part of the horizontal Line ; then draw $\mathbb{C V}$ and $\mathbb{C} L$ paraller to the feveral Sides of the Square, which will produce $V$ and $L$ for the vanifhing Points of thofe Sides.---This being premifed, let us now compleat the Reprefentation, from the Side $a b$, which is given.--Through $L$, draw vl parallel to the horizontal Line, and make L1 equal to the Diftance LJ of the vanifhing Line vl ; then from L draw Lines through a and $b$, and continue them at pleafure; and from 1 , (which is the vanifhing Point of the Diagonal) draw a Line through $b$, cutting $L d$ in $d$; then from $d$, draw $d$ c parallel to $a b$, which will compleat the Face a b cd.. Again, from a and d draw Lines to V, and from d draw another Line to C , cutting aV in g ; and from L

[^28]बraw a Line through $g$, cutting $d V$ in $f$; which finifhes another Face : Finally, from c draw cV , and from f draw a Line parallel to dc, which will cut $c V$ in $e$, and thereby compleat the whole. Reprefentation.

Again, When the End adfg ftands in fuch a Manner that the Diagonal af is perpendicular to the Ground; in this Cafe, the Angles at $\mathbb{E}$, made by the Sides of the Square with the horizontal Line, will be each equal to 45 Degrees; and therefore $C$ is the vanifhing Point of the Diagonal of the Square : But if the End be fituated like the Square B, then draw a Square, repofing upon one of its Corners on the horizontal Lines, in the fame Manner: as, yout fuppofe the End of the real Cube to be fituated upon the Ground; after which, draw Lines from $\mathbb{C}$, parallel to the Sides of that Square, and then its Diagonals will cut the-vanifhing Line V L, continued, in the proper vanifhing Points of thofe Lines; thus, $F$ and H are the vanifhing Points of the Sides of the Square, and.G. is the vanifhing Point of one of its Diagonals.
EXAMPLE 2. When the Cibbe refts; upon one Edge ab; that is Fig. ${ }^{6}$. oblique with the Picture.
Continue ab to its vanifhing Point: and from $\mathrm{Z}^{\text {l }}$ draw a Line to the Eye E, with which make a right Angle; then draw EH, and through H draw $\mathbb{E}_{\mathrm{E}} \mathrm{E}$, perpendicular to the horizontal Line, and then is $\mathbb{U r} L$ the vanifhing Line of the End adef. Again, let H be the vanifhing Point of the Diagonal ed; then, by making HUa and HL equal to the Diftance HE, we fhall have the vaniming Points of the Edges ad, $\mathrm{df}, \mathcal{E} c$. and if from $\mathbb{O}$ and Il we draw
 cdfg. Again, find the vanifhing Point D of the Diagonal bd , and from $D$ draw a Line through $b$, then from $L$ draw Lines through $a$ and $b$, and then will Ld cut D d in d , and thereby give ad for the Edge ad; therefore, from d draw dil, which compleats one Face. In like manner, from a and d draw Lines to $\mathbb{A}$, and from d draw dH , cutting a $\mathfrak{e l}$ in e ; then from $L$ draw a Line through $e$, cutting $d \mathfrak{t}$ in $f$; and then we fhall have compleated another Face adef: Finally, from fdraw a Line to $\mathrm{mb}_{\mathrm{I}}$, and from c draw a Line to $\mathbb{U}$, which finifhes the whole. Figure.

Example 3. When the Cube fands upon one Corner, as'a. 'Fig. 37: Let us fuppofe the Cube to ftand in fuch a manner, that a Line paffing through the upper and under Corners will be perpen-
dicular to the Ground; in which Care, a Plane aceh, that paffes through thofe Corners, will be perpendicular to the Ground alfo, and confequently, its vanifhing Line will be perpendicular to the horizontal Line. And let us, moreover, fuppofe, that this vanifhing Line, as KI, paffes through the Center of the Picture.-Any where, at pleafure, draw a Squave $z$ and its Diagonals; then; rupon the horizontal Line, draw a Perpendicular AE, at pleafure allof: and at the Point.A, with the horizontal Line, make an Angle of 54 Degrees, and draw AA; then at A make another Angle CAD of 36 Degrees, and draw A.C; and make A.H equal to the Diagonal of the Square $z$, and. $A C$ equal to one of its Sides; thea draw CE parallel to AH, and EH parallel to AC: So fhall we have a Plane ACEH, which may be confidered as the original of the Reprefentation aceh; whofe longeft Side $\mathrm{a} h$ is inclined to the Ground at an Angle of 54 : Degrees, and whofe fhortert Side is inclined to the Ground at an Angle of 36 Degrees; which together make a right Angle; that is, one Angle of a Square. Now, having made thefe neceffary Preparations, let a be the Corner upon which it fftands.---Set off the Diftance of the Eye CE, and draw KI through C; then from E, draw EK parallel to AH, (Fig. X) and EI parallel to AC, (Fig. X) cutting KI in I and K ; then are $I I$ and $K$ the vanifhing Points of the Plane ace $h$. Again, through Kidraw atie parallel to CE, which will give the vanifhing Line of the upper Face cdef; and fince $K$ is the vanifhing Point of its Diagonal ce, therefore, by making K ut, and K I, each equal to the Diftance EK, we fhall have the vanifhing Points of the Sides c , $\mathrm{cd}, \mathcal{E}_{\mathrm{c}} \mathrm{c}$. by which means that Face may be compleated. Then let ce be the Diagonal given.--From c draw cila, chl, and from and at draw Lines through e, which will produce the Face propofed. Again, having two vanifhing Points of and I, of the Face a bod.---Draw OXI, which is its vanifhing Line; and by finding the Center O , and its Diftance $\mathrm{O} \mathbb{E}$, together with its vanifhing Point $H$ of the Diagonal bc, the Face abced may be compleated. The fame may:be faid of the other Face a cfg. The Figure $K$ is drawn in fuch a manner, as to thew all the Faces of a Cube in the above : Situation.

Leet us now, without any Regard to a particular Situation of the Cube, fuppofe ab one Edge given, difs vanifhing Point, and dFI its vanifhing Line; and let C be the Center of the Pi\&ture, and CE its Diftance.--Find the Center and Diftance of the vanifhing Line

and then will I be the vanining Point of the Edge ac；and by finding the vanifhing Point $H$ of the Diagonal $b c$ ，the Face $a b c d$ may be compleated．For，from the vanifhing Point $I$ ，draw a Line through the Center of the Picture，and continue it at pleafure，（as $I K$ ；）then from C draw a Line perpendicular to IK，and make CE． equal to the Diftance of the Eye；then draw IE ，and at E make a right Angle；then draw E K，cutting IK in K ；finally through： K draw a Line parallel to CE，which will pafs through the vanilhing Point $\mathbb{U}$ ，and produce the vanifhing Line $\mathfrak{c}$ 监 of the Face cdef． Again，from 祭 and I draw 县I，which will give the vanifhing Line of the other Side acfg．

I have dwelt the longer upon this laft Figure，as it is a very curious Example，and，as it were，opens the Way to the Projection of all the regular Solids．

ExAMPIE 4．To find the Reprefentation of a regular Tetrabedron，Fig．${ }^{88}$ ． repofing upon one of its Faces．
This alfo may be done eafieft by finding a perpendicular Plane which is fuppofed to pafs through the Middle of the Body，as a de．－－ Now in order to find the Inclination of the Sides of this perpendi－ cular Plane，draw an Equilateral Triangle A GF，and divide the Side GF into two equal Parts；and draw AE；then at E，with the Diftance EA，defcribe an Arc；and at A，with the Diffance AG， defcribe another Arc，cutting the former Arc in D；then draw AD $D_{2}$ ED；and then will AD be the Inclination of the Edge ab，and is 55 Degrees；and ED is the Inclination of the Edge ed，and is 70 Degrees．Having thus got the Inclination of the above Edges，the next thing is to find the Reprefentation of the Face $a b c$ ，the va－ nifhing Points of whofe oblique Sides are H，L．－－－－Bifect the Angle HEL，and draw EC；then is C the vanifhing Point of a Line that will divide the Side bc into two equal Parts；and therefore C is the vanifhing Point of ae，that is，of the Bottom of the perpendicular Plane ade．Again，through C draw $\mho \mathbb{D}$ perpendicular to the horizontal Line，and continue it at pleafure；and then is $\mathfrak{U} D$ the vanifhing Line of the Plane ade：Then at E，the Diftance of the vanifhing Line ${ }^{(1)}$ ，make an Angle with the Line CE equal to 55 Degrees，and draw EUA；and then is the vanifhing Point of the Edge ad．Again，make another Angle at E of 70 Degrees， and draw ED；and then is D the vanifhing Point of the Side de； by which means the Plane a de may be compleated；and by draw－ ing $b d$ ，and $c d$ ，the whole Figure will be finifhed．－－－Or it may be dons．
done by making the Figure ADE in fuch a manner that $\angle \mathrm{E}$ may be parallel to the horizontal Line; for then, by drawing E厅, and ED, parallel to AD and DE, the vanifhing Points $\mathfrak{A}$ and $D$ will be produced.---Or this Figure may be found by having only the Inclination of the Edge ad, which, we obferved before, was an Angle of 55 Degrees. Thus, make an Angle of 55 Degrees at E, and draw Eix ; then fince $H$ is the vanifhing Point of the Edge ab , and $\mathbb{U}$ is the vanifhing Point of the Edge ad, therefore by drawing $\mathfrak{d} \mathrm{H}$, and continuing it at pleafure, we hall have the vanifhing Line $\mathbb{U}^{2}$; and by finding the Center and Diftance of that vanifhing Lime, and making two Angles of 6o Degrees each at $\mathbb{C}$, we fhall have the vanimhing Points of all the Edges of the Side abd; and confequently, by joining d c, the Figure will be compleated.---What is faid of the vanifhing Line ay, may be faid alfo of the other vanilhing Line 4.1.*

Fig. 39. Exampie 5. To put a Canted Cube + into Perfpective, reffing upon one of its Square Faces.
Let a c be one Edge of its under Face, A its vaniming Point, and H the vanifhing Point of another Edge of the under Face; that is, let A and $H$ be the vanifhing Points of a Square that lies flat upon the Ground.---Through H draw FI perpendicular to the horizontal Line, , and make $H \mathbb{C}$ equal to the Diftance of the vanifhing Line FI. 'Then at $\mathbb{C}$, on each Side of the horizontal Line, make an Angle of 45 Degrees, and draw $\mathbb{C} F, \mathbb{C} I$; then from $F$ and $I$, draw FA, IA; and then is IA the vanifhing line of the Face ab.c.d; therefore, by finding the Center and Diftance of this vanifhing Line, and by that means the vanifhing Point K of the Diagonal $b c$, the Square abcd may be compleated.---In like manner, it is eafy to fhew, that FI is the vanifhing Line of the Face g; GM the vanifhing Line of the Face $h$; $\mathbf{z}$ N the vanifhing Line of the Face $b ; B D$ the vanifhing Line of the Face $f$; F \& the vanifhing Line of the Face $\mathrm{k} ; \mathrm{FN}$ the vanifhing Line of the Face i; Hel the vanifhing Line of the Face $m$; and $\mathfrak{W}$ the vanifhing Line of the Face e.

[^29]B 2.




Example 6. To put a double Crofs into Perfpective.
Let $C$ be the Center of the Picture, CL the horizontal Line, $\mathbb{F i g}_{8} 40$. and LC the Diftance of the Eye; and let L be the vanifhing Point of the Diagonal of a Square whofe Side $A B$ is parallel to the Pic-ture.---Now fuppofe A B to be the neareft Edge of the Bottom of the Crofs, ---Continue $A B$, and make $B D$ equal to it; then from $B$ draw BC, and from D draw DL, cutting BC in a; and from the Points A, B, a, draw the Perpendiculars An, Bm, ak, and make $B m$ equal to thrice $A B$; then draw $n \mathrm{~m}$ parallel to $A B$, and finifh the Top $n k$; after which, draw 3 and si parallel to AB, and continue 23 and 51 on both Sides, at pleafure; then make 34 and 25 equal to 32 , and draw 4 q and 56 parallel to nA ; then from 4, 2, 5, draw Lines to $C$; and through i draw p 8 parallel to 45 , and from 6 draw 6 C , then from 8 draw 87 , parallel to 56 ; which finifhes a fingle Crofs. Again, from C draw Lines through $1,2,3$, at pleafure; and from $L$ draw a Line through 3; cutting $c C$ in $c$; then draw ce parallel to 32 , and from $e$ and $c$ draw ef, cd , parallel to 12 ; then from d draw df parallel to ec; finally, from the Corner 8 draw a Line to $L$, which will cut ${ }_{i} C$ in $b$, and by that means give the farthert Corner $b$; from whence the whole Figure may be compleated.

I have added this laft Figure, to fhew the vaft Eafe and Expeditioufnefs of this Method, preferably to any yet made publick; and I fhall have a further Ufe for it in another Place: However, I will juft obferve, that there are no more than four Lines in the whole Operation but what are a Part of the Reprefentation itfelf.

## C H A P. III.

## The Practice of Perfpective abbreviated.

S E CT. I.<br>General Rules, Ec.

IN the laft Chapter I have given fome general Rules, and have explained them by the moft ufeful Examples, fo that the wholePractice of Perfpective might be deduced therefrom by any one who will confider them with a proper Attention: But left their Application, in general, thould not appear fo eafy as could be wifhed, (and to fpare the Learner as much Trouble as poffible) $\mathbf{I}$ fhall, in the firf Section of this Chapter, collect all the Rules together in their proper Order; and then, in the fecond Section of this fame Chapter, I fhall apply them more particularly to common Practice.

2ig. 41. 1. Having one Line AB given, wofofe vanifting Point is out of the Pieture, from thence to drawe another Line CD, which Jhall. tend ta the fame Point.
From the Point A draw A.C at pleafure, and at any convenient Diftance, (the farther the better) draw BD parallel to A C.---Now let AC reprefent the Corner of any Building, and C the Top of it; and let it be required to draw a Line, as CD, which is fuppofed parallel to AB.---Make B6, which is below the horizontal Line, in the fame Proportion to 6D, which is above the horizontal Line, as $\mathrm{A}_{2}$ is to 2 C ; that is, fuppofe 2 A two equal Parts, and 2 C three equal Parts; then divide the Space 6 B into two equal Parts, and make 6D equal to three fuch Parts; and then draw CD, which will, if continued, vanifh into the Point E. And by proceeding in the fame Ratio, any other Line may be drawn, either above or below the horizontal Line.

1. When 'tis below the Horizontal Line, to draw a Line, as I5, which tends to an inacceffible Point E.
Let A B be the given Line.-..-Draw A 2, B6, parallel to each other, and divide them in the fame Proportion; thus, let $\mathrm{A}_{2}$ be divided into two equal Parts, then divide B6 into two equal Parts. alfo, and a Line, as I 5 , which is drawn through the Points 1,5 , will vanifh inta $E$.
2. When it is above the Horizontal Line, as 48.

Here CDD is the Line given, and 'tis required to draw 48 in fuch a manner that it fhall cut off one-third of the Plane C 26 D . Divide 2 C and 6D each into three Parts, and draw 48 ; which is the Thing propofed.---Thefe Rules are applied to Practice in the 3 th, 21 ft , and 23 d Figures.
II. To make one Line equal to anotber Line given.

This may be done by giving a Line, as a c, parallel to the hori- Fig. te. zontal Line.---For let cH be an indefinite Reprefentation, Part of which is to be cut off equal to ac. From H, fet off HI equal to HE, and draw a $I$, which will cut off bc equal to a c.---And fuppofe it were required to find the Length of any Line which is in an inclined Plane, then the very fame Method is to be ufed. Thus, in Fig. 44, let V be the vanifhing Point of $a b$, and VL, the vanifhing Line of an inclined Plane, in which ab is fuppofed to lye. ---Find the Center C of that vanifhing Line, and its Diftance CE; then make $V_{2}$ equal to the Diftance $E V$ of the vanifhing Point $V$, and from a, draw ac parallel to the vanifhing Line $V L$, and make it equal to the propofed Length, and then draw $2 c$, cutting aV in b ; which will give ab equal to ac . Again, to make one Line, df , equal to another Line cb , which is given. Let H be the vanifhing Point of $c b$, and $C$ the vanifhing Point of $d$ f.---From the Points H and C, defcribe the Arcs IE, LE; from c draw a c parallel to the horizontal Line; and from I draw a Line through b, cutting acin a; then is acequal to cb : For continue ac 'till it cuts Cd in d, and make ed equal to ac ; then draw e $L$, which will cut off df equal to $\mathrm{c} b$.*

## III. To cut off a Line in any given Proportion.

Let a Line be drawn parallel to the horizontal Line, and conti- Fig. 43. nued at pleafure; and let it be required to cut off ac equal to three Feet.---Thro' one End, as a, draw a Line 34, cutting the Bottom of the Picture in 4, and the horizontal Line in 3 ; then fet off 3 I equal to the Diftance ${ }_{3} \mathrm{E}$; and from I draw a Line through 2 , cutting the Bottom of the Picture in e; then from e fet off three Feet upon the Bottom of the Picture, (as eh) and draw h I, cutting acinc; fo will ac be equal to three Feet.---Again, To cut off an oblique Line, as ab, equal to three Feet.---Set off the Dif.

[^30]tance 3 I , and from e and h draw $\mathrm{e}, \mathrm{h}$, , cutting $a_{3}$ in $a$ and b; then is ab equal to three Feet.--Or this may be done without taking the whole Diftance ${ }_{3} \mathrm{E}$ : Thus, take half the Diftance, as 3 2, and divide the given Line ac in the fame Proportion, that is, into two equal Parts, and draw $f_{2}$, which will cut off ab equal to ac. After the fame Manner, a Line may be divided into any Number of Parts, or be made of any given Length; for by fetting the real Proportions upon the Bottom of the Picture, it may ferve as a general Scale for regulating the apparent Size of any Perfpective Reprefentation : Thus, a b , or a c , may be divided into three Parts each, or three Feet, by dividing eh, and then drawing Lines to 1 , in the above Manner.---Thefe Rules are particularly applied to Practice in the 18 th, 1 gth, and 20 th Figures.
What is here faid in regard to the cutting off Lines in any given Proportion, when thofe Lines vanifh into the horizontal Line, is equally applicable to Lines in all Kinds of inclined Planes,
Fig. 44. For let VL be a vanifhing Line of an inclined Plane, and V the vanifhing Point of $a$ Line $a b$ in that Plane.--Continue $V a$ to the Bottom of the Picture in B, and from B draw AB parallel to the vanifhing Line VL, and continue it at pleafure; then, upon thisLine $A B$ fet off the feveral. Meafures, as if it were the Bottom of the Picture, and confider VL as the horizontal Line, C as the Center of the Picture, and CE as its. Diftance; and then the Operation will be the fame as in the laft Figure.--This alfo is applied to Practice in the $34^{\text {th }}$ Figure.
IV. Having Retched-in tbe propofed Size for an Object upon tbe Picture, to prove whether it be diminiJhed in Proportion to its Diftance.
Fig. 45. Let a b reprefent the Height of an Object.----From any Point as H , in the horizontal Line, draw Lines through the Extremities a and b of the Figure, and from C , where the loweft Line cuts the Bottom of the Picture, draw the Perpendicular CB; then make this Line a Scale, the fame as if it was the Bottom of the Picture; and that will fhew whether the Figure be in Proportion for the Place it poffeffes in the Picture, or not: 'Thus, fuppofe the Height is 20 Feet; then $\mathrm{g} f$ is too much, and cd too little. Thus again, fup-
Fig. 46. pofe the Object to be a Houfe, I fay, its Height ab may be proportioned to its Diftance by the above Rule: Thus, continue the Edge ab upwards at pleafure, and from the vanifhing Point $C$; of the End, draw Cb through the Top of the Roof, cutting ab in b ; then is $a b$ the whole Height of the Houfe; therefore from $a$ and $b$ draw
draw two Lines parallel to the horizontal Line, and continue them at pleafure; then any where from the Bottom of the Picture draw 2 Perpendicular, as $1_{2}$, and to any Point 5 in the horizontal Line draw 15 cutting $3^{\text {a }}$ in 3 , and from 3 draw the Line 34 parallel to 12 , cutting $\mathrm{b}_{4}$ in 4 ; finally, from 5 draw a Line through the Point 4, cutting 12 in 2; and then is 12 the real Height of the Houfe, which being meafured by a Scale of Feet, will thew whether the Houfe be in proportion, or not, to its Diftance...--This is likewife applied to Practice in abcd, Fig. 18.
V. To find the Length of any Reprefentation by Calculation only.

Let $A B$ be a real Line whofe Reprefentation is fought, and Fig. 47: CE the Diftance of the Eye; which, in this Cafe, is parallel to AB.---Make B a in the fame Proportion to Ca , as AB (the real Line) is to CE, the Diftance of the Eye: Thus, let AB be two Parts, and CE three Parts (or, if you pleafe, fo many Feet; then divide AC into five equal Parts, and the Reprefentation Ba of BA, will be two of thofe Parts; that is, as 2 is to 3 : For draw AE, which will determine the Reprefentation of $A B$, as in the 5 th Figure. In like Manner, ec is to cC , as 2 is to 3 ; or, if you pleafe, ec is to the whole Line eC, as 2 is to 5 . And fo alfo for any oblique Line, as DF: For Dd is to dL, as DF is to LE. Or this may be determined, without taking the whole Diftance, by Analogy. Thus, half the Diftance of CE, as CI, and half the Length of $A B$, as $A$ 2, will come to the fame thing; for draw 2.1, and it will pafs through the Point a.---The next Figure is a farther Explanation of the fame thing, but by a different Method.

## VI. To find the Difance of the Picture from baving two vani/hing Points of a Square given.

Let V L be the horizontal Line, C the Center of the Picture, and Fig. 49 $\mathrm{V}, \mathrm{L}$, the vanifhing Points of the Square.--Divide VL into two equal Parts, in A, and with the Diftance AV defcribe the Semicircle VEL; then from C draw the Perpendicular CE, cutting the Semi-circle in E; and then is CE the Diftance of the Picture. For draw VE, LE, and they will make a right Angle at E.

## VII. To find tbe vanißbing Lines of any inclined Plane, and tbieir proper vanifling Points, together with the Center and Difance of thofe vanijbing Lines.

I. When the Plane is inclined to the Ground, but bas fome of its Sides parallel to the Picture, like acde. Fig. 3 I .
In this Cafe, the Rules are laid down and fully explained by that Figure.
2. When
2. When the Plane is notionly inclined to the Ground, but bas all.its Sides ollique with the Picture, like abde. Fig. 34.
In this Figure alfo the Rules are fully explained. And thefe are all the Rules which are neceffary in common Practice, as I have obferved before; neverthelefs, to affift the Curious, I have added the following Figure.

Fig. 5o. VIII. Having given one Side of any inclined Plane, at pleafure, together with its vanifling Point, and the vanifling Line of that Plane, thence to determine the wobole Reprefentation.
Let VU be a vanifhing Line given, and fuppofe $V$ the vanifhing Point of one Edge of a Square, (as ae, Fig. 36) and let C be the Center of the Picture, and CE its Diftance.----From $V$, the given vanifhing Point, draw a Line through the Center of the Picture, as $V D$, and continue it at pleafure; then from $C$ draw. CI perpendicular to VD, and make CI equal to the Ditance CE, and from V. draw VI, then from I draw I c perpendicular to VI, cutting $V D$ in $c$; and parallel to IC draw $L U$, cutting $V U$ in $U$; then is UL the vanifhing Line of a Plane perpendicular to that Plane, whofe vanihing Line is $V U$; that is, $V U$ and $L U$, in this Figure, are the fame as $\mathbb{H} L$ and: $\mathrm{Z} L$ in the 36 th Figure, where the Plane abcd is perpendicular to the Plane adfe. Again, Suppofe the Angle which another Plane made with the Plane whofe vanifhing Line is VU, was a different Angle (fuppofe 60 Degrees) and it was required to find its vaniming Line.-Then, as before, draw a Line VD through the Center C of the Picture, and find the vanifhing Line LU of a Plane perpendicular to that Plane, whofe vanifhing Line is $V \mathrm{U}$; after which, continue $c D$ at pleafure, and make $c D$ equal to the Diftance Ic of the vanifhing Line $L U$; then draw DU, with which make an Angle at $D$ equal to 60 Degrees, and draw Diz; finally, draw $V$ I $;$ and then will $V$ 最 be the vanifhing Line propofed.---Again, to find the Center and Diftance of a vanifhing Line.---From the Center of the Pinture, draw a Line perpendicular to any vaniming Line, which will give the Center of that vanifhing Line: Thus, CH and Cc are perpendicular to $V U$ and $L U$, and therefore $H$ and $c$ are the Centers of thofe Lines.---Again, for the Diffance of a vanifhing Line.---Upon the Perpendicular Cc, and at the Center C, draw another Perpendicular C I, and make CI equal to the real Diftance $C E$; then draw I $c$, which will be the Diftance of the vanifhing

Line $L U$, which being transferred into Cc continued, as CD , will give the proper Diftance to be work'd with.
IX. The following is a Method for finding the Reprefentation of the Fig. 5rid Plan of any Building, Ecc. when the Diftance to be work'd with is not greater than from the Center to one Corner of the Picfure, as CE:
Draw CE; perpendicular to which, draw K through C ; which confider as the Bottom of the Picture. Under K draw out the real Figure in its proper Situation, as A B C; then from the tranfpofed Place of the Eye, draw Lines parallel to the feveral Sides of the Figure, which will give $H$ and 1 for the vanifhing Points of thofe Sides, and which are to be tranfpofed into the horizontal Line, as H and L ; after which, draw the Perpendicular CM , and from M fet off the feveral Diftances CF, CG, $\mathcal{E} c$. upon the Bot= tom of the Picture; and then, by drawing Lines to the proper vanifhing Point of each Line, as in the Figure, the whole Reprefentation may be compleated, exactly in the fame Manner as if the original Figure had been drawn out under the Picture.
In the next Place, I fhall fhew how to determine the Appearance of thofe Sorts of Objects which moft frequently occur in common Practice; for this will explain more fully the Ufe of the preceding Rules, and at the fame Time, will thew the Shortnefs and Expeditioufnefs of this Method of Perfpective. And as I have by former Examples, fo I fhall likewife, in the next Section, make ufe of fuch Objects as are fimple in their Parts, and of the moft general Ufe. To explain my felf more fully. A Pedeftal, for inftance, is but one Part of an Order in Architeटture, and the Idea we have: of it is, of its being the Bafe, or Support, of a Column; but by enlarging the Idea of a Pedeftal to that of a large fquare Building, enriched with Mouldings, $\mathcal{E c}$. we may then confider it as fuch a Building; and therefore, we may conceive, that the fame Rules by which the Appearance of a Pedeftal is determined upon the Picture, will ferve for finding the Reprefentation of any Building which is fimilar to it. In like manner, as to the Situation of $\mathrm{Ob}-$ jects which are perpendicular to the Ground, (fuch as the Walls of Buildings, and the like) they muit be either perpendicular to the Picture, parallel to the Picture, or oblique with it; as we have fhewn before: And therefore, one Example in each Situation, adapted in a general Manner, will be of much more Service than ten thoufand different Schemes by way of Examples; for the one
fixes our Attention to a particular Set of ufeful and general Ideas; but the other diffracts the Mind with Confufion and Obfcurity.

The fame Arguments will appear equally true, if we apply them to the particular Parts of any Building, fuch as Columns, Mouldings, and the like. For, firft, in regard to Columns; by this Method, we have no Regard to Plans, Elevations, Éc. and therefore, it matters not where we begin the Operation, whether at the Top, the Bottom, or at the Middle of it; fo that one Rule alfo in this Cafe will appear to be univerfal: And in refpect to Mouldings, they muft be either plain or curvilinear, either above or below the Eye; and therefore, one Rule in either Cafe will be fufficient for our Purpofe. The fame may be faid of every other Example in this Section; but what has been faid already, will, I hope, be fufficient to explain the Senfe of the following Figures, and to filence any Objections which may be made againft my not having fwell'd my Work with more ornamental Schemes, or, as they are generally called, Curious Examples.

The firft Example which I fhall produce, is the TvSCAN Pedestal, in order to thew how to find the Reprefentation of ftrait Mouldings, when they are either parallel, perpendicular, or oblique with the Picture, or when they are either above or below the Eye. In the $5^{2 \mathrm{~d}}$ Figure one Side is parallel to the Picture, the other perpendicular to it; and in the 53 d Figure, both Sides are oblique with the Picture.

S E C T. II.
The foregoing Rules of Perfpective more particularly applied to common Practice.

1. To put a Tuscan Pedestal into Perfpective.
2. When one of its Sides isparallel to the Pieture, then the other Side will be perpendicular to it; fo that one Rule will do in both Cajes.
Fig. 52. ET AB reprefent the Bottom of the Plinth in Front.---Now, from this one Line AB, the Appearance of the whole Pedeftal may be found: For continue $A B$ at pleafure, and draw a 1 ine IK perpendicular thereto, and make IK equal to the Height of the Pedeftal; then, upon IK, draw the Capital and Bafe in their proper Proportions: This being done, continue Lines from each Moulding,


XII.

4 I.

46.

51.




Moulding, which will form feveral Rectangles, and thereby divide the Planes $1234,5.6 .78$, into a Sort of Net-Work; then, by putting thefe Planes into Perfpective (as in the Figure) we fhall have fufficient Guides for drawing all the Mouldings. But to be more particular in the Operation.---Make $A D$ equal to $A B$, and cut off $A E$ equal to $A D$; from whence the Plinth may be com-pleated.---In like Manner, for the Die.---Draw the Diagonals upon the Top of the Plinth, and any where upon the Edge hg, fet off the Projection of the Plinth, as I 9 ; then draw one Line from I to $C$, and another Line from 9 to $L$, which will give the Projection I 2 ; then, if you draw a Line through 2 parallel to hg , it will cut the Diagonals a h, bg, and give the Corners of the Front Side ; and if you draw a Line from $b$ to $C$, it will cut the Diagonal $\mathrm{d} c$, and give the further Corners; therefore, by drawing Perpendiculars from a, $b, c$, we fhall have all the Edges of the Die which can appear in this Situation.--As to the Height of the Die, or the Height of the feveral Mouldings, they may be found by drawing a Perpendicular from the neareft Corner of the Reprefentation, as $B \mathrm{~B}$, and transferring thereon the feveral Heights from IK, as in the Figure; then by drawing Lines from the feveral Points upon BH (which meafures the Height of each Moulding) to the vanifhing Point of the Diagonal ah, we fhall have thofe Heights tranfferred unto the Edge amof the Die; thus a i and km are the Heights of the Bafe and Cornice; fo that by finding the Appearance of the Planes 1234,5678 , and drawing the Mouldings therein, and by drawing the triangular Planes at the Corners, we may finifh the whole Reprefentation with the utmoft Eafe and Expedition.

But before we begin to draw out any Object in Perfpective, we muft firft confider, whether the Whole, or only a Part of it, is to appear; and muft fketch out the Size we intend it fhall be of, or, at leaft, give one Line for its greateft Dimenfion. Thus, if the whole Pedeftal is to appear, then give AB, which is neareft the Eye, and call it the utmoft Length of the Plinth: But if only the Top is to appear, then give HO, and call it the utmoft Extent of the Cornice ; then, by cutting off Or equal to OM , that is, equal to OH, we fhall have the Depth Or of the Cornice, ©c. from whence, and with the Affiftance of the Plane 5678 , (which is found exactly in the fame Manner as the Plane I 234 ) we may compleat the whole that is wanted.

Now, in order to do all this, it is neceffary that the Artift fhould (as was obferved before) be able to apply the preceding Rules with. the greatef Readinefs; particularly That which teaches how to cut off one Line equal to another Line given.
2. When both Sides are oblique with the PiEture.

Fig. 53. In this Figure, let A be the neareft Corner of the Plinth, AC, AB, the Length of two Sides A.G, A.F, and Ak the Height of the whole Pedeftal properly divided; (that is, like BH in the laft Figure.)---Cut off AF, AG, equal to AB, AC, and draw the Plinth and the Diagonals upon the upper Square; then draw 1 b parallel to AB, and make it equal to the Projection of the Plinth; then cut off $I c$ equal to $I b$, and from $c$ draw Lines to $H$, and from I draw a Line to C, (the vanifhing Point of the Diagonal 12) which cutting each other in 2, will give the Edge 27; and by drawing Lines from 2 to their proper vanifhing Points $L$ and H , they will cut the Diagonal kg , and thereby give the other Corners of the Die, as in the Figure.--- For the Height of the Mouldings; draw Lines from 4 and 5 to C, which will cut 27 in the Points required; by which means the triangular Planes aik, $\mathrm{fb} \mathrm{g}, \mathcal{B} c$. may be found, and from thence the Mouldings may be compleated.

Here alfo, if we want only the upper Part, we may begin at the Point 8, making $8 \mathrm{E}, 8 \mathrm{D}$, each equal to the Length of the Cap, §c. then, by finding the Plane 5678 , as in the former Cafe, we fhall have fufficient Guides for compleating the Figure.

Here let us obferve, that when the Pedeftal has one Side parallel to the Picture, then the Plane 1234 , (Fig. 52) which is a Guide for the Moulding, may be begun any where upon the Edge a $b$ : But when it is oblique with the Picture, then we muft begin from the neareft Corner, as a; and by attending to the Figures, we may conceive, that in the firft Cafe, the Mouldings in the Directing. Plane, are like the Ends of Mouldings cut off fquare; but in the latter Cafe, they are like Mouldings cut off at what is called the Mitre Joint. And from hence we may alfo obferve, that all the Difficulty in putting Mouldings into Perfpective, lies in finding the little Planes I 234 , छc. and therefore the Reader fhould confider them attentively before he proceeds any farther.

## II. Of Circular Mouldings, Esc.

The Method for determining the Appearance of Circular Mouldings, is much the fame as that for finding the Reprefentation of fraight Mouldings, viz. by imagining a Plane to pafs through the Mouldings in a perpendicular Manner, and then putting that Plane into Perfpective : As in the two laft Figures.

1. To put a Tuscan Base into Perfpective.

Give one Line for the Width of the Plinth, and draw out the Fig. 540 proper Projection of the Mouldings, and the Plane ABCD; then cut off the oblique Side equal to the Front, and compleat the Plinth; after which, draw the Diameters and the Diagonals upon the Top of the Plinth, as in the Figure; and then draw the Reprefentation of a Circle for the Seat of the lower Torus. Again, for the Bottom of the Shaft of the Column; from the Center H of the Column, draw the Perpendicular $\mathrm{HL}_{2}$ and from a, where the Diameter ae cuts the Plinth, draw ad parallel to H ; then make ad equal to the Height of the Mouldings AD, and from d draw a Line to $C$, which will cut HL in I; then will I be the Center of the Square for the Bottom of the Column : Therefore, upon the upper Edge of the Plinth, and from the Point a, make a I equal to the whole Projection of the Mouldings; then cut off ab equal to a I , and draw bc parallel to ad , which will give the little Plane for the Mouldings; within which draw the Mouldings; and then we fhall perceive that $c$ is the Middle of the neareft Edge of the upper Square: Therefore, through c draw a Line EF parallel to the Edge B I of the Plinth; then from H draw a Line through the Center I , cutting EF in E, and then is c E half theWidth of the Square : Therefore make cF equal to CE , and from thence compleat the Square, and within it draw the Reprefentation of the Circle, as in the Figure: Finally, from the Extremity of each Circle draw the two oblique Lines 2, 3, which together with the little Plane for the Mouldings, $\mathcal{E} c$. will be fufficient Guides for compleating the whole Bafe, as was propofed; which is evident by infpecting the Figures 57 and 59.

As for making Columns, Eic. of any given Proportion, or at any Diftance; the Rule for cutting of a Line in any given Proportion, in the 43 d Figure, is fufficient for that Purpofe.
2. To put a Tuscan Capitai into Perfpertive.

Fig. 55. Let K be the Center of the Square for the Bottom of the Capi-tal.---Through K draw a Line AB parallel to the horizontal Line at pleafure, and from $f$ and $e$ of the Bafe, draw Lines parallel to the Axis HI of the Column, cutting the above Line in E and F; then is EF the Diameter of the Column: Therefore diminifh it in its proper Proportion, as 34 ; then is 34 the Diameter of the Neck of the Column: Therefore with the Line 34 draw the Appearance of a Square, and in that Square draw the Reprefentation of a Circle as before directed; fo fhall we have a Guide for the under Part of the Capital. Again, make KI equal to KF, (that is, equal to Half the Diameter of the Column) and through I draw.GH parallel to $A B$, then make IG, IH, each equal to $K A$, or $K B$, (that is, equal to Half the Diameter of the Top of the Abacus) and then with the Line GH draw the Appearance of another Square, which will reprefent the Top of the Abacus: Finally, from C draw a Line through I, cutting the Edge 56 in a, and from a fet off a 1 for the Projection of the Capital, and draw the little Plane abcd for the Mouldings, as before: From whence the remaining Part of the Capital may be compleated, as in the 56 th and 59th Figures*

I fhall juft mention a Method for finding the Point where the. Diagonal of a Square will be cut by a Circle infcribed in that: Square; which may be of ufe in this and fome other Cafes. It is
Fig. 54. this: Divide the Edge BG into feven equal Parts; then fet one Part from each Corner, as $\mathrm{B}_{4}, \mathrm{G} 5$, and draw Lines to C , which will cut the Diagonals in the Points required. I do not fay, this, is mathematically exact, but, I prefume, it is near enough for the: intended Purpofe.

Thefe are the moft fimple, as well as the moft general Methods I can think of for mix'd Mouldings; and I believe any Perfon who is but tolerably fkilled in Drawing, will find them fufficient for his Purpofe, upon all Occafions.
3. To find the Reprefentation of a Corinthian Capital.

Fig. 62. Let AB be the Diameter of the under Part of the Capital, and let ca be the Center, or Axis, of the Capital, properly divided for the Height of its Leaves, Volutes, and Abacus.---From the Line

[^31]$\mathbf{A} B$, which is given, find the Appearance of a Square, in which, draw the Diameters and Diagonals, and then the Reprefentation of the Circle; which will determine the Places for the Stalks of the great Leaves, as reprefented by the Dots: Again, through o draw bd parallel to AB , and make ob, od, each equal to half the under Part of the Abacus; with which Line bd, draw the Appearance of another Square, and divide it like the Plan 1234 , Fig. Z. of the under Part of the Abacus, and then draw the Reprefentation of it, as in the Figure: Again, through a, draw another Line parallel to bd, and make it equal to the upper Part of the Abacus; then, by finding the Reprefentation of the Square abcd, Fig. Z, we may draw the Appearance of the upper Part of the Abacus, and from thence compleat the Abacus, as in the 60 th Fi gure: Finally, find the Middle of each Face of the Abacus, as. $\mathrm{n}, \mathrm{e}$, and draw Lines $\mathrm{n} \mathrm{I}, \mathcal{E}_{\mathrm{c}}$. to the correfponding Points at the Bottom of the Capital; then find the Height of the Leaves by drawing Lines from $C$ through the Dots in $c a$, 'till they cut in in 2 and 3 ; after which draw the Bafket; then, by a nice Eye, compleat the Capital; beginning as is exemplified in the 60th Figure.---The Lines drawn from the Corners of the upper and under Square, will ferve as Guides to prevent our giving the Leaves too much Projection. In Fig. 6I, the Capital is compleated, and the Figures X and Z are added to explain the Thing more fully; one of which is the Plan, and the other Half the Profile of a Capital.

Here it is neceffary to take Notice, that upon Account of bringing the Diftance of the Pi\&ture within the Compafs of each Plate, and to make the Figures as large as poffible, fome of them have not that agreeable Shape which could be wifhed; but if the Reader will choofe a greater Diftance, and follow there Rules, he will find every Objection of this kind, that may arife, immediately vanifh.

## III. Of Couvmns parallel and oblique with the Eye.

1. Let it be required to find the Appearance of two Columns in Front, Fig. 63. and let a b. be the Diameter of the under Part of the Plinth, and c the Center of the Column.
Continue $a b$ at pleafure, and any where upon it, as at $A$, draw a Line $A B$ perpendicular to $A b$; upon which Line fet the feveral Heights for the Bafe, Capital, Entablature, छc. then from $c_{2}$ the Center of the Column, draw a Line $c d$ parallel to $A B$, and
and from the feveral Divifions upon $A B$, draw Lines parallel to the horizontal Line, which will cut cd, and give the Heights of the Bafe, Capital, and Entablature. Now, having got the feveral Heights, we are to confider cd as the Axis of the Column, (that is, a Line which paffes through the Middle of it) and then at every Dot make a Square, equal to the Diameter of that Part of the Column, $E_{c}$. which that Dot ftands for: Thus ab is the under Part of the Plinth, and by means of H, and the Diagonal 12, we may compleat the firft Square. So alfo, $r$ is the Square for the Bottom of the Shaft, $p$ for where the Column begins to diminifh, $t$ for the Top of the Shaft, and $v$ for the Top of the Abacus; and therefore, having got thefe feveral Squares, we fhall have fufficient Guides for compleating a Column of any Order. Again, for the other parallel Column ex.--From c to e, and upon the Line $A B$ continued, fet off the Diftance which the Center of one Column is from the other, and draw ex; upon which, fet off the feveral Heights, as before, from the Line $A . B$, and then find the feveral Squares, as before directed.

## 2. For oblique Columns.

Upon $A B$ continued, fet off the Diftance which the Centers of thofe Columns are from the Center of the Corner Column, and draw a Line from e to $C$; then cut off em, en, equal to ef, eg, and from the oblique Sides of the Square e, draw Lines to $\mathbf{C}$; from whence the other Squares $m$ and $n$ may be compleated, as before.--For their Heights,--Draw mo and nq , parallel to ex, and from x draw xC , which will give their feveral Heights.

Now, if we would put an Entablature over the Columns; then the Height of the Architrave, Freeze, and Cornice, may be drawn from their refpective Divifions, upon the Line AB; and the Appearance of the Mouldings peculiar to each Part, may be found by the Rules already laid down for that Purpofe. Or, they may be found thus,--Let $\mathrm{agb} f$, in the Figure Z , reprefent the upper Part of the Capital $x$ belonging to the corner Column; and we want to find the Corner of its Entablature; that is, the Corner of the lower Facia, or Freeze :--Set off the Projection of the Capital from n to e , and draw a Line to C , which will cut the Diagonal ab, and give the Corner of the Facia; fo that by drawing the Line 1 from the Dot in the Diagonal, we fhall have the Corner propofed. In like Manner the Projection of the Cornice may be found. Continue $7^{\circ}$ (which meafures its Height) at pleafure, and make

- h equal to the real Projection of the Cornice; then through $h$ draw a Line from C , and from H draw a Line through o ; and where it cuts. Ch continued in $i$, will be the Corner of the Cornice, $\Xi^{\circ}$ c.

From hence: then it is evident, that any Number of Columns may be drawn with the greateft Eafe and Expedition, let their Situations be what they will; and from hence alfo we may obferve, that any Part of a Column may be immediately produced upon the Picture in the very Place in which it. is intended, without drawing, out the Whole, or any other Part than that which is really to appear..

Here it may not be improper to take Notice of what: we have obferved in Chap. VI. Book I. concerning the Reprefentations of parallel Columns, which, we obferved, would grow bigger and bigger the further they are removed from the Center of the Picture; and to point out a Method to be practifed by thofe who are fatisfied with the Reafons upon that Head; and by that Means, to give all the Columns an agreeable Shape. It is this :----Firft find Fig. 63: the Reprefentation of that Column which is neareft the Center of the Picture; as ex; then fet off the Diftance for the Centers of the other Columns, and draw the Squares for the Plinth, Capital, $\mathcal{O}^{\circ} c$. and then, upon each Side of the Axis, fet off at the Bottom of each Column Half the Diameter of the corner Column, and at the Top of the Column fet off Half the Width of the Neck of the corner Column : Finally, draw Lines from thence, fo as to diminifh the Column in a proper Manner ; and thereby we may make all the Columns that are parallel to the Eye of the fame Bignefs. As to the great Projection of their feveral Bafes, they will not look at all prepofterous, if they are done by any one who has but a tolerable Eye for Drawing, and is careful in taking a proper Diftance for the Eye.

## IV. Of Stairs parallel and oblique.

1. For parallel Stairs.

Let $A B$ be the Length of one Step, $\mathrm{B}_{1}$ its Height, $\mathrm{B}_{4}$ its Fig. 64 : Depth, C the Center of the Picture, and CH the Diftance of the Picture.---Find the firit Step by the Height BI, and its Depth B 4; then make 4 O equal to 4 B , and iD equal to 1 B ; by which means the upper Step may be found. Now, by continuing BO, and BD, and by dividing them properly, any Number of Stairs may be determined. Or, if only thofe above the Eye are to be feen, then
then begin with the Line $a b c$, and proceed in the aforefard Manner.

> 2. For OBIICUESTAIRS.

Fig. 65. Let $A B$ be the Length, BO the Depth of two Steps, BD the Height of two Steps, and $C$ the Center of the Picture.---By means of the Points $\mathrm{H}, \mathrm{L}$, cut off $\mathrm{Ba}, \mathrm{BF}, \mathcal{E}_{\mathrm{c}}$. equal to $\mathrm{B} A, \mathrm{BO}, \mathcal{E}_{\mathrm{c}}$, then by making BD equal to the Height of the Steps, they may be compleated, as in the Figure : And after the fame Manner the Stair above the Eye is to be found.

> V. Of an ARch and PEDIMENT.
I. For the ARCh .

Eig: 66. Let A be the Corner of the Arch.---Set the feveral Divifions for the Width and Center of the Arch upon A B, and for its Height upon the Edge A 1 , and draw Lines to C; by which means the parallelogram cdef may be found, which will be 2 Guide for drawing the Arch.

> 2. For the PEDIMENT.

Let abD be the Fitch of the Pediment, and CD the Diftance of the Picture.---Find the vanifhing Points $V$ and $L$, and draw Lines from thence through the Top and Bottom of the Cornice ; which will interfect each other, and give the Reprefentation of the Pediment.

## VI. Of Houses parallel and oblique. <br> i. For parallel Houses.

Fig. 67. Let AB, and BF, be the Length and Depth of the Houfe, BD the Height of the Walls, and C the Center of the Picture.--Draw the vanifhing Line V L, and find the vanifhing Points V, $L$, of the Roof; then draw V I parallel to the horizontal Line, which will be the vanifhing Line of that Part of the Roof which fronts the Eye. And the Roof which covers the Pediment bde is found by drawing the perpendicular ab , and, from the Extremities thereof, Lines to $C$ and $V$, which will give a Triangle $a b c$, whofe Side cb is the Top of the Roof.---Or the Height of the Roof 34D, may be found by continuing BE , and making DE equal to the propofed Height.

## 2. For Houses that are oblieue.

Fig. 68. In 'this Figure $A B$ is the Depth, and $A C$ the Length of the Houfe, VI is the vanifhing Line of the End, and V, $I$, are the vanifhing
nifhing Points of the Roof; the vanifhing Line of the Pediment, $\mathcal{E}_{\mathrm{C}}$. is V , and its vanifhing Points vand 1 ; the Length of the Roof over the Pediment is found by means of the Triangle $a \cdot b c$, as before.
VII. To puit the Injude of a Room into Perjpective.

Let ADFE be a Picture upon which the Infide of a Room is ifig. 69 ; to be drawn.---Here C is the Center of the Picture, and AB the Length of the Room.---Set off the Several Divifions from A to B, and cut off the feveral Spaces upon $A_{1}$ equal to thofe upon AD; then fet off the Heights upon AE, and draw Lines to C ; which will be fufficient for our Purpofe.

If any Reprefentation of this Kind is to be drawn upon a Wall, fo as to make a Deception like the Continuance of a Rooin, Care muft be taken to choofe a proper Diftance, and to make :the Height of the horizontal Line exactly equal to the Height of the Eye, viz. about five Feet fix Inches. If the Wall be too large for any Diftance that can be taken within the Room, then fome other Subject muft be painted upon it, fuch as will admit of its being divided into Frames, Compartments, or the like: And the fame may be faid in regard to Cielings; for, in either Cafe, if the Diftance be an improper one, all the Reprefentations will have a bad Effect.

Thefe Examples are the moft general I can think of; and I flatter myfelf, that they will be found fufficient to anfwer every Defign which can be propofed in common Practice: But if there Thould appear any Difficulty in applying the aforefaid Rules upon fome extraordinary Occafions (as when the Defign confifts of many Parts, or when it happens that any of the vanifhing Points are out of the Picture) then the beft Way will be to draw out the whole Defign, by. Way of Model, in a fmall Compafs, upon Paper, and from thence, by a proper Scale, or by Net-Work, to transfer the whole unto the real Picture; for then the moft diftant wanifhing Points may be very eafily come at. Or in many Cafes, the real Picture may either be laid flat upon a Floor; or elfe have Rules made to fix upon the back Part of the ftraining Frame by Screws, or fome fuch Contrivance, whereby, and with the Affiftance of fmall Twine fixed upon Pins at each vanifhing Point, we may produce almoft every Reprefentation which can be defired.

The three following Figures I have not only given as Examples in Perfpective, but have attempted to difpofe each Object in fuch a Manner as to produce agreeable Shapes, Effect, Ecc.--The firft reprefents a Variety of Figures tending to various vanifhing Points in the horizontal Line, below the horizontal Line and above it; amongft which, are the five regular Solids; and the whole together, contains all the Rules and Principles of Perfpective. The next Figure is a View of Framlingbam Caftle in Suffolk a Place of great Antiquity, and formerly the Seat of the Howards, Mowbrays, $\mathcal{O}^{c}$. which is produced in this Place as an Example of a Building that tends to feveral vanifhing Points upon the horizontal Line only: And the laft Figure is an Example of a Lande Akip, by a very great Genius in that Way.




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## CHAP. IV.

## Of the Parallel Picture, fuch as Cielings, or the like; or what is ufually called, Horizontal Perfpective.

THIS Kind of Perfpective is extremely eafy, becaufe little more is required to be known than what has been already taught in Sect. II. Chap. II. of this Book; viz. How to find the Appearance of Objects which are fuppofed to lie upon the Ground. For moft Objects which are drawn upon Cielings, are fuppofed to be perpendicular to them; and therefore, the Rules for determining the Reprefentations of Objects in this Manner, are exactly the fame as thofe for determining the Reprefentations of Objects which lie flat upon the Ground, in the perpendicular Picture; and confequently, the Rules which ferve in one Cafe, will ferve in the other alfo.

The firft Things to be confidered in thefe and the like Reprefentations, are, the Diftance of the Eye, and the Center of the Picture.---As to the Diftance of the Eye, that is unalterable, becaufe the Picture is fixed; therefore, if the Cieling be fo large, or fo low, as to fubtend too great an Angle at the Eye; that is, if the longeft Dimenfions of the Cieling be much greater than the Diftance at which the Spectator is to look at it; then, in this Cafe, the Cieling fhould be divided into Compartments, which may ferve as Frames for the intended perfpective Reprefentations: And we muft be always careful, when we take the Height of any Cieling from the Floor, to deduct the Height of the Spectator's Eye therefrom, which is ufually about 5 Feet 6 Inches. And in regard to the Center of the Picture, the general (and, I believe, the beft) Method has been, to fix it in the Middle of the Picture, unlefs any Thing prevents the Eye from feeing it conveniently from that Place; becaufe then there will be a Uniformity of the Parts, which will agree with each other, and be more likely to deceive the Eye.

And it is to be obferved, that in thefe Kinds of fixed, or immoveable Pictures, the Spectator fhould always fix his Eye directly againft the Center of the Picture ; for otherwife the Reprefentations will not have their defired Effect.

Now, in order to draw any Piece of Perfective upon a Cieling, the beft Way feems to be this, viz. Take the Dimenfion of the Cieling, and make an exact Calculation of the Diftance and Heighat of the Eye; then draw out the intended Defign upon a large Piece of Paper, by Way of Model, and from thence transfer it unto Canvas, with the Addition of Colouring, Effect, $\mathcal{E} c$. and finally, from thence draw it upon the Cieling, by Net-Work.

## 1. To draw upon a Cieling a Deception, which, viewed from a proper Point, Sall appear like the Sides of the Room continued upwards.

Fig. 73. Let ABFD be a Cieling drawn upon Paper to a certain Scale ; 74. and let E be the Eye, EC its Diftance, and C the Center of the Picture. Now, let it be required to make the Cieling appear as if the Sides of the Room were continued upwards equal to the Length AB.---Through the Center of the Picture draw E1, E2, parallel to $A B$, which may be confidered as the horizontal Line; then draw Lines from the Corners $\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{D}$, to the Center C , and make CEI equal to CE; by which means A a may be cut off equal to the given Length $A B$, and confequently, from thence all the Reprefentations may be compleated, as in the Figures: Thus, ab being drawn parallel to $A B$, gives the Side $A B a b$, and ad being drawn parallel to $A D$, compleats the Side $A D a d, \mathcal{E}_{6}$....In the 73 d Figure, the Center lies out of the Middle of the Picture; but in the 74 th Figure, the Eye is directly in the Middle.

Here we may obferve, that if Lines are drawn thro' the Center of the Picture parallel to the Sides of a Room, then thofe Lines may be confidered as fo many horizontal Lines, and may be made ufe of accordingly. Thus, $\mathrm{E}_{1} \mathrm{CE}_{2}$ will ferve as a horizontal, or vanifhing Line, for all Objects which can lie upon the Planes $\mathrm{ABab}, \mathrm{DFdf}$; and $\mathrm{ECE}_{3}$ will ferve as a horizontal Line for any Objects which can lie upon the Planes ADad, BFbf.---By turning the Figures we may conceive this very clearly; but in the next Figure it is more fully explained.

In this Figure I fhall fhew how to find the Reprefentation of fuch Objects only as may occur in common Practice, fuch as Columns, Pilafters, Arches, and Windows. And firft of Columns.

Fig. 75. Let the Circles $\mathbf{H}, \mathrm{I}$, reprefent the Ends of two Cylinders, and let $\mathrm{E}_{2} \mathrm{CE}_{4}$ be the vanifhing Line of the Plane $\mathrm{ABab}, \mathrm{CE}_{3}$ the Diftance, and C the Center of the Picture.--About each Circle
$\mathrm{H}, \mathrm{I}$, defcribe a Square, and make $\mathrm{CE}_{4}$ equal to $\mathrm{CE}_{3}$; then draw a Line from each Corner of the Square to C ; and then, by means of the Point $\mathrm{E}_{4}$, a Parallelopiped may be made of any Length, which will be a Guide for compleating the Cylinder; as is fhewn in Figure 29 of this Book. Now, by the fame Method, the Appearance of Columns may be determined, with this Difference only; that three Squares muft be found as Guides inftead of two; that is, one for the Bottom of the Column, another where it begins to diminifh, and the. Third at the Neck of the Column:
3. To find the Reprefentation of Two Pilasters..

Let F and G be the Ends of the Pilafters.---From each Corner draw Lines to C, and, by means of the Point E. 2, cut off each Pilafter to its proper Length.
4. To determine the Appearance of a Seuare Object which lies oblique with the Picture.
Let 5 be one Corner, 57 the given Length, and $E_{2}, E_{4}$, the vanifhing Points of the Sides.---From the Corner 5 draw Lines to the above vanifhing Points, and cut off 56 equal to 57 ; from whence the Figure may be compleated.

On the oppofite Side DEde, I have finifhed thefe Reprefentations with Shadows, $\mathcal{E}^{2}$.

## 5. To put an ARch into Perfpective.

Let KM be the Width of the Arch, Mh the Height to where the Arch fprings, and hi the Height of the circular Part; and let $\mathrm{E}_{\mathrm{I}} \mathrm{CE}_{3}$. be the vanifhing Line, $\mathrm{CE}_{2}$ the Diftance of the Picture, and $\mathbf{C}$ its Center.---From $K$ and $M$ draw Lines to $C$, and cut off $\mathrm{Mn}_{\mathrm{n}}, \mathrm{no}$, equal to Mk , hi, then draw the Parallelogram nopq, which will be a Guide for drawing the Arch. Again, for the Depth of the Opening,---From K draw a Perpendicular to DA, and make it equal to the propofed Depth; and from its Extremity z draw a Line to C ; then from q draw a Line parallel to Kz , which will cut Cz , and thereby give the proper Depth; do the fame on the other Side; then draw the bottom Curve for the other Side of the Arch, parallel to the upper Curve, as in the Figure; and fo will the Reprefentation be compleated.
6. To find tbe Appearance of a WINDOW, the Top of which we. will Juppose to be even with the Top of the Arch, and to be two Diameters in Height.
Set off the real Width Lf, and its Height Lg, and from the: Points L, $f, g$, draw Lines to $C$; then continue the Line op to $r$, which will give the Top rs, and from E I draw a Line through the Corner $r$, cutting $g t$ in $w$; then from $w$, draw $w v$ parallel to $r t$, which compleats the Window rsox. The Depth is found in the fame Manner as the Depth of the Arch, viz. by the Perpendicular L.

On the oppofite Side to this alfo, are the above Figures wholly compleated.
7. To pat a Cornice into Perfpective:

Draw out the Projection, $\mathcal{E} c$. of the Cornice, about which deFig. 76. fcribe the Plane A B CD; then put that Plane into Perfpective, as FGHI; from whence all the Mouldings may be determined, as in the Figure.
8. Toput a BAse and Capital into Perfpective.

Fig. 77. For the Bafe,---Altho' nothing more than the Plinth can in general be feen by the Eye, yet I have here given a Method for determining the whole Projection.---Let $A B$ be the Diameter of the Plinth, and BF the Height of the Bafe; make a Square with $A B$, and from $B$ draw a Line to $C$, and cut off $B D$ equal to $B F$, and from D draw DC parallel to $A B$, and with DC make another Square ; then divide AB into eight equal Parts, and one of thofe Parts (according to Gibbs) is the Projection of the Mouldings : * Therefore, make BI equal to one of thofe Parts, and from I draw a-Line to C, cutting the Edge of the fartheft Square in 3; then from 3 draw a Line parallel to $C D$, and fet off the Diftance $3 D$ upon the other three Sides of the parallel Square; then, by drawing Lines through thofe Points, we fhall have a Square equal to the Diameter of the Column; and from thefe two Squares the whole Appearance is to be compleated.----The Figure G reprefents it as finifhed.
9. To put a CAPITAL into Perfpective.

Let AB be the Diameter of the Bottom of the Capital, 12 the Diameter of the Abacus; 13 the Height of the Capital, and B 2 the Projection of the Capital.----Set off B b equal to B2, and make the
the Square abcd; then draw $1 C, 2 C$, and cut off $1 D$ equal to 13 ; then draw DF parallel to $A B$, and with DF make a Square; finally, from the Corner of one Square draw Lines to the correfponding Corners of the other, as in K ; then fhall we have fufficient Guides for compleating the Capital.

## 10. To put the Human Figure into Perfpective.

Having made a Defign of the Figure, defcribe the Frame about it, as ABCD, and then reticulate it in a proper Manner; after which, put that Frame and the Reticulation into Perfpective, as abcd , which will give all the Forefhortnings, as in the Figure.

I am very fenfible, that 'tis impoffible to give Rules for putting: the Human Figure correctly into Perfpective, and that the greateft Part muft be left to the Judgment of the Artift ; yet the above Hint may be of fome Service in defigning Figures for the above Purpofes. So likewife, as to the Size of Figures which are to be feen at a confiderable Diftance; I know of no Rules by which they can be correctly determined; and therefore, in fuch Cafes, the beft Way is, to fketch out feveral Figures of different Sizes upon the intended Picture; then, by furveying them from the Point of View ${ }_{3}$. the Eye will immediately inform the Artift which is of a proper Proportion.

In Book I. Chap. IV. Sect. 3, we have given fome general Rules from Mr. Hamilton, for drawing any Perfpective Reprefentations upon vaulted Roofs, Domes, or other uneven Surfaces; and therefore, if the curious Reader would inform himfelf of that Kind of Perfpective, he muft refer to thofe Figures, where this Article is confidered at large; which will fufficiently explain the 7 8th and 79 th Figures, fince they are thofe Rules applied to Practice. But as I have faid very little upon drawing a Dome, $\mho_{0} c$. upon a flat Cieling, and as the Operation is quite Mechanical, I fhall therefore introduce it in this Place.

In order to find the Reprefentation of Domes, $\S c$ c. it is neceffary to draw out the Plan, and Half the Elevation, of the Defign which we intend to reprefent, to a proper Scale, upon Paper: Thus, let the 80 th Figure be the Section, or Half the Elevation, of the intended Defign; and let the two outward Circles,* and the fmall *Fig. $e_{\text {zo }}$ Squares and Circles within them, reprefent the Plan of it: From which two Figures we may perceive, that the Defign confifts of cight Columns upon Pedeftals, with an Entablature, in the Corinthian Order; that thofe Columns are fuppofed to ftand againft
a per-
a perpendicular Wall A e, Fig. 80 ; and that the Dome is a Semicircle, and begins to fpring from the Top of the Cornice. Now; having drawn out the Plan and Elevation, as above directed, the Reprefentation of any Defign may very eafily be determined in the following Manner.
II. To drawe upon a flat Cieling the Reprefentation of a Dome *.

Having given the Elevation and Plan, choofe the Center and Diftance of the Picture. Thus EC, Fig. 8I, is the Diftance of the Picture, and C is its Center; that is, EC is the Diftance at which the Eye is to view the Dome when painted, and directly under C is the Point from whence the Eye fhould be placed to look at the Picture. Now, this Point C being taken out of the Picture, will give a greater Length for the Columns, $\mathcal{E}_{\mathcal{C}} c$. and will prevent fome Confufion, which would be occafioned by placing the Center within the Picture. Thefe neceffary Points being fettled, let us next defrribe the Parallelogram ABCD about the Elevation, Fig. 8o, and then draw Lines parallel to $A D$, from the feveral Heights $g$, $f, \mathrm{e}, \mathcal{E}_{\mathrm{c}} \mathrm{c}$. as in the Figure: After which, from the Center of the Picture C, (Fig. 8I) draw CD perpendicular to $E C$, and from $E$ draw $E A$, parallel to $C D$; then from $A$ draw $A D$ parallel to $E C$, and continue it beyond $D$ at pleafure; and then will EA be a Line for the Plan, and AB a Line for the Elevations: Therefore, from the Point A fet off the feveral Meafures from AD, Fig. 80, which are the Meafures of the Plan; and from $A B$ of the fame Figure, fet off the feveral Diftances, which are the feveral Meafures for the Elevations: Thus Ad, Fig. 8r, is the Width of the Plan AD, and AB the Height of the whole Defign, properly divided for the Height of the feveral Members, which may eafily be conceived by comparing the two Figures 80 and 81 . Having proceeded thus far, the next Thing is to put the Elevation into Perfpective, as the 8Ift Figure; where $\mathrm{Abcd}, \mathcal{E} c$. is the Reprefentation of $A B C D$, Fig. 81. This is done by drawing Lines from $A$ and $d$ to the vanifhing Point $C$, and then drawing other Lines from the feveral Divifions upon A B, which cutting A C in correfponding Points, will give the apparent Depth of each Part ;

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from whence the whole levation may be reduced into Perfeetives, as in the Figure. Having proceeded thus far, the next Thing: (and indeed the Principal of all) is, to defcribe feveral Circles. each from a different Center, and each of a different Diameter; which is done thus : From the: feveral Divifions, as n, m, k, draw Lines parallel to AE, cutting d $C$ in $r, p, o$, then from $d, r, p, o$, draw Lines parallel to $A B$, cutting the Perpendicular $C D$ in $\mathrm{I}, 2,3,4$; then is I the Center of the outward Circle, and 1 D : (which is equal to Ad) is its Radius; therefore, defcribe the outward Circle, and from the fame Center defcribe the fecond: Circle, and then, within thofe two Circles, draw the Plan, as in. the Figure:. Again, for the Height of the Pedeftal ; draw $56, \mathrm{r} 2$, parallel to $A B$, cutting $C D$ in 2 and 6 ; then is 2 the Center, and' 26 the Radius of that Circle which governs the Heights of the: Pedeftals. In like Manner; 3 is the Center of the Circle which: limits the Length to where each Column begins to diminifh, and: 4 is the Center of the Circle for the Nofe of the Cornice; and foof the reft : All which may be made very familiar by drawing out the Figure. As to the Returns of the Pedeftals and Mouldings, they all vanifh into the feveral Centers of thofe Circles which determine their Heights: Thus 2. is the Center of the Circle for the Height of the Pedeftals; and therefore, the oblique Sides of thofe Pedeftal's terminate in that Point. And as to the Ornaments which may be drawn upon the Dome, they alfo are to be determined in the fame Manner; as will be evident by a very little Attention to the Figure, and by applying thefe Rules to Practice in a larger Scale than this upon the Plate...-.The 8oth Figure is the Reprefentation more nearly compleated, and between each Co-lumn I have introduced a Pannel to fill up the Vacancy, and to give a Hint how to introduce Ornaments proper for this Kind of: Reprefentations; for whether Figures, Feftoons; or any other kind of carved Ornaments, are intended ; by infcribing Squares about each, and by dividing them into fmaller Squares, we may reticulàte each Cell, which will be fufficient for forehortning all Kinds, of Ornaments.

## CHAP. V.

## The Per'spective of Shadows, goc.

## S E C T. I.

THIS Part of Perfective has been very little attended to by moft Writers upon the Subject, and yet it is very neceflary to be known, and very ealy to be underftood; for it is Built upon the fame Principles as the Perfpective of Objects, and, therefore, is deducible from the fame Rules. But I would not be underfood to mean, that the Shadow of every particular Object upon the Picture is to be determined in the following Manner; no; my Intention is, only to give fome general Principles, in order to explain the Reafon and Nature of fuch Shadows as are neceffary in the Arts of Defign; by which means the Artift will form a general Idea of the Perfpective of Shadows, and will be the better qualified to difpofe them in a Picture.

Shadows are either projected by the Sun, or elfe by a Candle, Torch, or fome fuch luminous Point. But fince thofe produced by a Candle, Ecc. are but feldom wanted, I fhall therefore principally have regard to fuch Shadows only as are projected by the Sun : Which may be reduced under the following Heads.

1. When the Light comes in parallel with the Picture.
2. When the Light comes from behind the Picture towards the Spectator.

## 3. When the Light comes from before the Picture.

In the firft Cafe, the Shadows will be parallel to the Bottom of the Picture; but in the fecond and third Cafes, fince the Light comes in oblique with the Picture ; therefore, both the Rays of Light, and the Shadows projected by them, will have their proper vanifhing Points ; and confequently the Shadows produced thereby will be oblique with the Bottom of the Picture. The vanifhing Point of the Rays of Light will be either above the horizontal Line or below it ; and thofe Points will always be in Lines drawn perpendicular to the horizontal Line *: And we may moreover obferve, that when the Light comes from behind the Picture, then the vanifhing Point of the Rays of Light will be above the horizon-

[^33]tal Line ; but when the Light comes from before the Picture, then the vanifhing Point of the Rays of Light will be below the horizontal Line: All which is exemplified in the following Figures. For in Figure 83 , the Light is fuppofed parallel to the Picture; therefore the Shadows are parallel : In Figure 84, the Light is fuppofed to come from behind the Picture, and $S$ is taken at pleafure for the vanifhing Point of the Shadow of the perpendicular Sides, and L for the vanifhing Point of the Rays of Light : In the85 th Figure, the Light is fuppofed to come from before the Picture; and here $S$ is the vanifhing Point of the Shadow, and $L$ the: vanifhing Point of the Rays of Light; which are both taken at pleafure.

From hence then, and from the following Examples, it will be obvious, that after having drawn out any Perfpective Reprefentation, the Shadow of it may be very eafily determined upon the Picture; therefore let us now apply what has been faid to Practice.

CASE 1. When the Light comes in parallel to the Picture.
Io find the Sbadows of the Objects A and B, which are fuppoled to be Fig. 83 : cafi upon the Ground.
Through all the Corners of the Bottoms of the Objects draw Lines parallel to the horizontal Line, and through every Corner of the Top of the Objects draw Lines parallel to each other for the Rays of Light ; and their Interfections with the loweft parallel Lines will determine the Appearance of the Shadows, as in the Figure: Thus a is the Shadow of A , and b of B .

From hence we may obferve, that fince EB is confidered as a Ray of Light, therefore EBD is its Angle of Inclination with the Ground; or, in other Words, with the Plane of the Horizon. And we may alfo obferve, that in Proportion as this Angle of Inclination of the Rays is greater or lefs, the Shadows will be longer or fhorter; which accounts for the Reafon why the Shadows of Objects are longer in a Morning and Evening, than when the Sun is at any confiderable Height above the Horizon : All which may be clearly apprehended by attending to the Figure; or by drawing out other Figures, and then giving different Inclinations. to the Rays of Light.

CASE 2. Then the Light comes in from bebind the Pieture.
Tig. 84. To find the Shadozers of the Objects A and a, which are fuppofed to be caft upon the Ground.
Take $S$ at pleafure in the horizontal Line, for the vanifhing Point of the Shadows which the perpendicular Edges caft upon the Ground (for as the Shadow lies upon the Ground, it muft vanifh into the horizontal Line;) and from this Point S, draw a Line SL perpendiculai to the horizontal Line: Then will SL be the vanifhing Line of the Rays of Light, and, confequently, fomewhere in this Line will be the vanifhing Point of thofe Rays. Now, in this Cafe, the vanifhing Point of the Rays is above the horizontal Line; therefore, take L at pleafure for that vanifhing Point, and from thence draw Lines thro' all the upper Corners of the Figures; then from the vanifhing Point $S$ of the Shadow, draw Lines through all the Bottom Corners; and their Sections with each other will be fufficient Guides for compleating the Shadows, as in the Figure: Thus, $L_{3}$ being drawn through 1, and $S_{3}$ being drawn through 2, will give the Point 3 for the Shadow of the Point 1 , and 23 for the Shadow of the Edge $12, \mathcal{E} c$.

Here let us obferve, that in order to determine any Shadow, nothing more is required than to find the Places of a certain Number of Points upon the Picture, which Points are to reprefent the Shadows of all the upper Corners of any given Objects: Thus, 3 is the Shadow of $I$, and 4 is the Shadow of A; therefore, draw 34, which is the Shadow of the upper Edge 1A; and fo of the reft.

## Casc 3. When the Light comes from before the Picture.

Fig. $\mathrm{s}_{5}$. To find the Sbadow of the Object A, which is fuppojed to be caft upon the Ground.
Here S is given for the vanifhing Point of the Shadow, LS for the vanifhing Line of the Rays of Light, and $L$ for their vanifhing Point; which, in this Cafe, is below the horizontal Line.-... From $S$ draw Lines through all the lower Corners of the Object, and from L draw Lines through all the upper Corners of the Object, as in the Figure ; and then their feveral Sections with each other will be fufficient for compleating the Shadow, as before.

From thefe two laft Figures alfo, we may obferve, that the farther the vanifhing Point of the Rays is taken from the horizontal Line, the fhorter will be the Projection of the Shadows; and the contrary, the nearer it is placed to the horizontal Line:: That is the nearer it is to the horizontal Line, the fefs is the Angle of Inclination which the Rays make with the Ground ; and the contrary. the farther it is from it. Again, by infpecting the two laft Figures, we may perceive, that when the Light comes from behind the Picture, the Shadows will be caft towards the Bottom of the Picture, and grow wider and wider continually, and the Front of every Object will be in Shadow; but in the laft Figure, the Shadows will be caft towards the horizontal Line, and will grow narrower and narrower continually, and the Front of every Object will be enlightened ; and therefore, thefe Kind of Shadows are the moft proper for a Picture, and confequently, deferve the moft Attention: For which Reafon, I fhall henceforth fuppofe the Light to come in this Direction only; and fhall now proceed to fhew how to determine the Appearance of Shadows as they are projected by different Planes, Esc.

In the laft Figure the front Side is parallel to the Picture, and the Method for finding the Shadow has been fhewn already; therefore proceed we to a Figure whofe Sides aie oblique, though the fame Rule is ufed in both Cafes.
To find the Shadow of the Object A, which is fuppoled to be caft upon Fig. 86: the Ground.
From the vanifhing Point $S$ of the Shadow, draw Lines thro the Bottom Corners, and from the vanifhing Point $L$ of the Rays of Light, draw Lines thro' the Top Corners, which (as before) will cut each other, and thereby give feveral Points, as Guides for compleating the Shadow.--If only the Shadow of the Top was required, then the Seats of each Corner muft be found upon the Picture; and from thence the Appearance of the Shadow may be determined: Thus 2 is the Seat of 1 , and 3 is its Shadow; and a is the Shadow compleated.
To find the Shadow of an oblique Object, wbich is Juppofed to be caff upon the Ground.
Here A is the oblique Side, S the vanifhing Point of the Shadow, Fig. 87. and $L$ the vanifhing Point of the Rays of Light.--From d draw dS, and from a draw aL; then will dc be the Shadow of the Perperdicular da; therefore by drawing bc, the Shadow will be compleated.

To find the Shadows of Objects when caft upon different Planes.
Fig. 88. 1. To find the Shadow of a perpendicular Object A, when it is cafl. upon a Plane inclined to the Ground, but bas Jome of its Edges,? as I 2, parallel to the Picture.
From the lower Corners of the Object A draw Lines to $S$, and from the upper Corners draw Lines to $L$, which would determine the Shadow of A, upon the Ground; but this Shadow being cut by the Bottom 12 of the inclined Plane, therefore Part of the Shadow will be caft upon it. Now, to find this Shadow, from $b$, (where the Line which is drawn from the loweft Corner of the Object A cuts the Edge 2n) draw bc perpendicular to the horizontal Line, cutting the inclined Edge 2 e , in e ; then from b and c draw Lines parallel to the horizontal Line, and from a (where ba cuts the Line drawn from the other Corner of the Object A to S) draw ad, which compleats the Parallelogram abcd; finally, from where the Edge $1 / 2$ cuts the Ground-Shadow, draw Lines through c and d, which will cut the Lines drawn from the upper Corners of A to L, and thereby determine the Length of the Shadow upon the inclined Face of the Object, as in the Figure.

## 2. To find the Reprefentation of a Shadow when it is caft by an inclined Object upon a perpendicular Plane enrs.

From $n$ and $e$ draw Lines to $S$ and $L$, which will give $m$ for the Shadow of e, and 2 mn for the whole Shadow of the Side 2 ne ; but fince 2 m is cut by the Edge $n \mathrm{r}$ of the Plane enrs, therefore, Part of the Shadow will be calt upon it; which Shadow is determined by drawing a Line from o (where 2 m is cut by nr ) to e; thus, noe is the Shadow which is caft upon the perpendicular Plane, and $20 n$ is the Shadow that is caft upon the Ground.

Fig. 89. 3. To find the Shadow of a perpendicular Object A, when it is caft upon a Plane B, that is every Way oblique with the. Picture, but is nevertbelefs fituated in fuch a Manner as to bave the vanifbing Line LP, of the perpendicular Side abc, pafs through the vanijbing. Point of the Sbadow.
From the upper and under Corners of A, draw Lines to $S$ and $L$, as before; then, from where the Ground Shadow is cut by the Edge a 1 , draw Lines to the vanifhing Point $P$ of the oblique Side B; which will cut the Lines drawn from the upper Corners of A, and thereby determine the Length of the Shadow.

## The Perspective of Shadows.

4. To find the Projection of the Shadore of a perpendicular Object A, Fig. 90. when it is caft upon an inclined Plane that is every Way obligue with the Picture.
Draw Lines from the upper and under Corners of $A$, to $S$ and $L_{\text {, }}$, then, from where $3 S$ cuts the loweft Edge 1 a of the farther Side of the inclined: Object, draw ab perpendicular to the horizontal Line, and continue the inclined Edge $1 b$ till it cuts $a b$; ther through a and $b$, draw Lines from the vanifhing Point of the Edge 12 , which will cut 4 S in c ; then, from c draw cd parallel to ab, which will compleat the perpendicular Plane abcd ; finally, from where the Ground Shadow is cut by 2 , draw Lines to b and d , which will cut 5 L and 6 L , and thereby give the Depth of the Shadow, as in the Figure.

Here let us take Notice, that as the Shadows of all Objects that are caft upon the Ground will vanifh into the horizontal Line, fo, for the very fame Reafon, the vanifhing Points of all Shadows which are caft upon any inclined, or other Plane, will be fomewhere in the vanifhing Line of that Plane, as was obferved in Figure 89 .

The 9 ift Figure is an Example of the Shadow of a cylindrical Object A, caft both upon the Ground and the Object B; and of the fquare Object C , which is caft upon the Ground and the Ob ject $D$; which, it is prefumed, wants no Explanation.

Before I conclude with the Shadows projected by the Sun, I fhall juft obferve, that altho' I have taken the vanifhing Points of the Shadows always within the Picture, for the Conveniency of Room in each Plate; yet, it is to be obferved, that, in general, the farther it is taken from the Picture the better.*

> Of Shadoros projected by the Candle, Esc.

Fig. 92.
In Shadows of this Kind, nothing more is required than to have the Luminous Point, and its Seat upon the Ground; for by drawing Lines from thofe Points through the upper and under Corners of each particular Object, the Shadow of that Object may be found, as in the former Figures; and only the Shape of the Shadows will be different; that is, they will grow wider and wider continually, the farther they are projected. I have given feveral Examples in this Figure, and have put every Line and Point that is neceffary in each Operation; which, it is prefumed, is fufficient for the Purpofe.

[^34]SECT.

## S E C T. H.

HAving fhewn how to determine the Appearance of Shadows, I might now proceed to the Confideration of Aerial Perfpective, $\mathcal{E}^{3}$ c. but as that is handled at large in the laft Chapter of the firft Book, the Reader is now referred to that: However, by way of Supplement to what is there advanced upon the Subject, I fhall. beg leave to make the following Obfervations. For, fince various. have been the Opinions about the Colour of Shadows, and as various the Methods purfued by Painters and other Artifts, I fhall therefore only offer a few Hints taken from Nature, which perhaps. may be of Service to the young Tyros in the Arts of Defign.

By Shadow then, in this Place, I mean the Colour of that Part. only of an Object, which is either turned from the Light, or is Fig. 94. wholly in the Shade. Suppofe, for Inftance, the Pillar W to be placed near this side of the Wall $b$, and fuppofe alfo, that the Rays: of Light came from the other Side of the Wall; then, it will be evident, that Part of this Object will be enlightened, and Part will be wholly in Shadow. Now, that Part which is wholly in. Shadow, is of the fame Colour as the whole Object would be of were the Sun not to fhine upon it; or, in other Words, 'tis of thefame Colour which the whole Object would be of in common light. From whence I infer, Firft, (allowing for the different Acsidents of the Sun's Light, the Air, © $\mathcal{E}_{c}$.) that the Shadow of the Pillar W, is the real Colour of that Object in common Light, but being oppofed to a fuperior Light, is, in comparifon of that fuperior Light, a Shadow. Secondly, That therefore the Colour of all Shadows muft be proportionably lighter or darker, as that Object to which it is a Shadow, is of a lighter or darker Colour. This I have explained in the following Manner. The Objects $W, W$, $I$ fuppofe to be White; the Object $Y$, to be Yellow; the Object $G_{2}$ Green; the Object R, Red; the Object B, Blue ; and the Object ib Black.---Here the fhadowed Parts of each particular Object are made darker and darker, in proportion as the Colours of the feveral Objects proceed from White to Black; which is evident by the Figure. Thirdly, Since then the Shadows of all the above Objects are nothing more than the Effects of common Light, compared with the Effects of the fuperior Brightnefs of the Sun; and fince Objects are as diftinctly feen by a common uniform Light, as they are in the Sun-fhine; therefore thofe Objects which are in Shadow, fhould be as highly finifhed, and their Parts as well made
out, in the Picture, as the Parts of the neighbouring Objects, which are in the higheft Light. And fourthly, from hence it follows, that the Shadow of every Object muft partake of the real Colour of that Object; and therefore, Black can never be the Shadow of White, nor of any other Colour than that of Black.

By thus ranging the Colours in their proper Orders, we may eafily conceive the Degree of Darknefs which is peculiar to the Shadow of each Colour. And if any one would moreover fatisfy. himfelf of the Truth of this, let him have a Number of fquare Pieces of Wood, painted of different Colours; then, by oppofing one Side to the Light, the Degree of Shadow will be very vifible.

And from hence alfo we may obferve, that the fronger the Light fhines upon any Object, the darker will be its Shadow; for in Proportion as the Sun fhines ftronger or fainter upon an Object, the Oppofition of the Light and Shadow will be greater or lefs; and confequently, the more perceptible will be the Shadow. And this accounts for the Blacknefs of Shadows by Candle or Torch-Light; becaufe the violent Oppofition between real Light and total Darknefs, together with the Faintnefs of the Reflections from the Smallnefs of the Luminary, munt produce that Effect.

From thefe Obfervations then it appears, that the Colour and Degree of Darknefs to each Shadow, is abfolutely neceffary to be known, and ought to be well underftood, in order to produce a good Effect in a Picture, or to reprefent any Object. as it appears in Nature. It is in this, and in a proper Diftribution of the Lights and Shadows in a Picture, that the Cbiara Obfcuro confifts; and it is this, and this only, which can give a Clearnefs to any Shadow, whether in a Painting, Print, or Drawing.

I: fhall juft offer a few Hints for determining the Appearances of the Reflections of Objects in Water, Ecc. and fo put an End to this Chapter.

The Reflections of Objects in Water, or any other tranfparent Medium, may be confidered, Firft, as to their Colour; and, Secondly, as to the Length of their Reflections. As to their Colour, If the Medium be very clear and tranfparent, the Colour of the Reflections is very near the Colour of the Objects; but in a thick or dirty Medium, the Reflection of an Object very fenfibly changes its Colour, and partakes more and more of the Colour of that Medium in proportion as it is more denfe and muddy, Book II.
'till, at laft, the Reflection will entirely difappear. And in order to make Water appear tranfparent (which is done principally by means of Reflections ) the Reflections fhould be as perfect as poffible.

## To determine the Reflection of any Object in Water.

Fig. 93. Let I be an Object ftanding upon a Hill, and a its Bottom; then continue the Sides of the Object downwards, at pleafure (as the prickt Lines in the Figure) and fuppofe c is where the even Ground cuts the Bottom of the Hill; then fet off cd equal to $\mathrm{c} I$, which will give the Length of the Reflection. Again, for the Object 3, which ftands upon the flat Ground; make the Length of the Reflection $f$ equal to that Object. The Object 4 is too far from the Water to be reflected by it; and the Reflections of the Objects $0, \mathrm{n}, \mathrm{g}$, which are floating upon the Water, are each equal to the Height of its peculiar Object. So alro as to the inclined Object 2 ; the Reflection of that muft have the fame Angle of Inclination with the real Object, and be of the fame Length, as in the Figure. From which it appears, that all Kinds of Reflections are very eafily determined; fince nothing more is required, than to fet off the perpendicular Height of each Object, downwards, upon the Water, Eoc.

What has been advanced upon Reflections, relates only to a fagnating Medium ; that is, a fill or fmooth Water, or the like; which is the fitteft for an Explanation of this Matter, and will be §ufficient for giving the Learner a general Idea of Reflections: But when either the Objects, or the Water, or both together, are in Motion, then, though the Reflections will be wavering and uncertain, yet the above Rules will be of great Service in fuch Cafes; and efpecially, if they are joined to the study of Nature.

I cannot conclude this Head without the following Quotation from Mr. Pope's Second Paitoral; which, to me, feems an inimitable Picture of Nature, and much to our prefent Purpofe.

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93.

Of REFIECTIONS in WATER:

I have now gone through with all I intended to advance upon the Subject of Perfpective, and wifh the Work may anfwer the Expectations, of my worthy Friends and generous Subfcribers; and that the great Pains, Labour, and Expence it hath coft me, may not prove in vain. I fay, that here I intended to have put an End to my Subject; but, by the Defire of fome particular Friends, I fhall take a. Tranfeript from Pozzo and Mr. Hamilton, in relation to SCENE-PAINTING; and then fhall add the different Methods of the mof confiderable Authors upon Perfpective; which may either ferve to divert or inftruct the Reader; and, at the fame Time, will fhew him, which are the preferable Methods, mine, or theirs, cither as to Eafe or Expedition.

## C H A P. VI.

## Of Scenography; or Scene-Painting.

SCENOGRAPHY is the Art of Painting upon feveral Planes, or Scenes, at different Diftances, and in various Pofitions with refpect to the Eye, in fuch a Manner that " all thofe different Scenes, when feen from one certain determi" nate Point, may correfpond with each other, and reprefent one "entire View of the Defign without Breaks or Confufion, as if " it were one continued Picture." This is Mr. Hamilton's Explanation of Scenography; who has handled this Subject in a very clear and comprehenfive Manner, both in Theory and Practice; and therefore, what I intend to offer upon it myfelf, fhall be principally an $A b f t r a c t$ from him. For, fince 'tis impoffible for me to treat it in a better Manner than he has done before me, I fhall therefore refer my Reader to his and Pozzo's * Books, if what I thall offer be not fufficient for his Purpofe; and in order to be as concife as poffible, I fhall fuppofe him to be acquainted with the Nature and Conftruction of Theatres in general, and that he only wants to know, how to draw fuch Reprefentations as are proper for fuch Places.

The Defign of Scene-Painting, is not only to decorate the Theatre, but to make that Part of it which lies beyond the Stage, appear much longer than it really is. This is effected by raifing the Floor to a certain Angle, by floping the Cieling, and by raifing the Scenes in fuch a Manner, that both Floor, Cieling, and Scenes, fhall be a Part of a hollow Pyramid, like LlOoNnMm, which, if continued, would meet in the Point $T$; and after having dimiFig. 95 . nifhed each Scene in its due Proportion, then by drawing thereupon the intended Defign, by the common Rules of Perfpective, fo that every Scene, when put in its proper Place, fhall appear as a Part of the general Defign.

As to the Inclination of the Floor and Cieling, and alfo the Ranging, and the Space between each Scene; thefe, as I obferved before, I hall fuppofe my Reader acquainted with, and therefore, Thall have Regard only to the drawing Perfpective Reprefentations upon Planes fituated in the above Manner.

[^36]Here APFD is that Part of the Theatre which is allotted for Fig 93. the Speetators, K GLM the Profcene, LMON the Curtain, a bcd the Aperture in the Curtain through which the Scenery is feen, MLml the Floor upon which the Scenery is placed, PQRS the farther End of the Theatre, E the Eye, EH its Height above the Floor ABDF, $h$ its Seat upon the horizontal Plane ef $Q P$, and T the Center of Contraction for the Scenes, Floor, $\mathcal{E}^{2}$.

Now let ABCD be a Plan of the Theatre, e e the Seat of the Fig 96. Curtain, 'ML the Opening of the Curtain, b b, ECc. the Grooves for the Scenes to flide in, H the Eye, and T the Point of Con-traction.--Here the Diftance CH of the Eye from the Curtain, is not taken fo great as in the laft Figure; for, was the Point of Sight placed at one End of the Houfe, then the moft ordinary Part of the Company would have the beft View of the Scenery; and therefore, about the Middle of that Part of the Houfe which is allotted for the Spectators, is thought the moft proper Place for the Eye; as in the Figure.

And having determined the Plan for the Scenery, and fixed the Point of Sight and Center of Contraction, let us next determine the Height of the Eye, and the Height of the feveral Scenes.
Let ABCD be a perpendicular Section of the Houfe in the Line OT (Fig. 96.)---Draw Lines perpendicular to HT from the Points H and T, 'till they cut the Line CS of the 97th Figure in the Points $h$ and $S$; then is h the Seat of the Eye, and $S$ the Seat of the Point of Contraction: Again, continue the parallel Lines through the Seats of the Scenes 'till they cut the Line CS in I, $1,2,3,4$, which will give the Diftance between each Scene; and from the Point I draw Ie, for the Inclination of the Stage, and continue it beyond T , at pleafure : Then, for the Height of the Eye and Point of Contraction, make Eh in this Figure equal to Eh in Fig. $95^{*}$, and draw ET parallel to CS, cutting IT in T; then is Eh the Height of the Eye, and T the Point of Contraction,

From hence it is evident, that fince the Floor Ie is fixed, the Point of Contraction muft be governed by the Height of the Eye. For let $x h$ be the Height of the Eye; draw a e parallel to hS, and then is e the Point of Contraction; and in Proportion as the Height of the Eye is greater or lefs, the Point of Contraction will be nearer or farither off, By this Method of varying the Height of

[^37]the Eye, great Variety of Scenery may be introduced; but how: far 'tis allowable to alter: the Height of the Eye in Scenes for the: fame Entertainment, muft be left to Experience to decide: However, this we may obferve, that it ought never to be above the: Middle of the Opening of the Curtain, (that is, above s in Fig.95) nor much below the Face of an Actor upon the Stage. And in. regard to the Point of Contraction; it is not neceffary to have it. upon the End of the Wall, at t, but it may, and ought in general, to be placed beyond it: For when it is placed at the End of the Houfe, then the Scenes will be too fuddenly diminifhed, and will have a difagreeable Effect, befides other Inconveniencies.

## Fig. 97.

Again, for the Height of the Scenes-- The Line rs is the perpendicular Section with the Curtain; and the Curtain being confidered as a Picture, therefore C is the Center of the Picture; and therefore, upon the Line is fet off the feveral Diftances from $s_{s}$. for the hanging Scenes and Tops of the fide Scenes; thus c is for the Tops of the Scenes, and ca for the Widths of the hanging Scenes; therefore from $\mathrm{S}, \mathrm{c}$, a, draw Lines to T , which will give: the Height of each Scene, $\mathcal{E}_{c}$. as in the Figure.
Fig. g6. The Side Scenes are made to project beyond the Line $M \mathrm{~m}, \mathrm{EBC}_{\mathrm{c}}$. which meafures the Opening of the Curtain ; and they mould be: brought fo forward upon the Stage, that a Line Hb , drawn from: the Seat of the Eye thro' the Corner of the firf Scene a, may meet: the fucceeding Scene in the Point $b$, where it is cut by Mm : For, by this Means, the Spaces between the Scenes will not be vifible ta many of the Spectators; but the whole together will appear like.
Fig. 97. one continued Picture. In like Manner, Lines drawn from the: Top Corner of each Scene, as b of the Scene fb, to the Eye, will give c a for the Width of the hanging Seenes.
Fig. g6.
Again, if Lines are drawn through the Points $f, g, h, i$, till: they cut the Line eD, then thefe Points $f, g, h, i$, will be the Projections of the Points I, 2, 3, D upon the Floor of the Stage; and confequently, the oblique Line ei, will, to the Spectators at H , appear to be equal to the Line eD; fo that the Back Scene in the Line ik, will appear to be as far from the Eye as the End of the Houfe CD; and, by that means, the Depth of the Theatre will appear to be much greater than it really is.

Having made thefe neceffary Preparations, we will now proceed to fhew how to draw the Reprefentations upon each Pair of Scenes, fo that the whole, when viewed from a proper Point, fhall appear as one continued Picture.

## To prepare a Pair of Side Scenes for Painting,

Draw a a at pleafure, which call the Line of Interfection that Fig. 9\%. the Scenes make with the Floor of the Stage; then from any Point. $c$, erect the Perpendicular cE, and from the Plan (Fig. 96) take the Diftances $d x, d x$, which the Scenes $x b, x b$, are from HT, and transfer them from $c$ to $b$; take alfo the Width of each Scene,
 downwards, and make cg equal to fi, (Fig. 97) and draw $f \mathrm{fh}$ parallel to $a a$ : Then are $a b$, da, the Seats of the fecond Pair of Scenes, and gc their Height from the horizontal Plane efQP, (Fig. 95.) And fo alfo for the Height of the Scenes;---From $a, b, d, a$, draw Lines parallel to $g E$, then take the Height $f b$ of the fecond Scene, (Fig. 97) and fet it from b to $i$; which gives the proper Height. Again, for the horizontal Line and Center of the Picture;--Take Id (Fig. 97) and fet it from g to $\mathrm{C}_{2}$ and through C, draw HL parallel to a a; then is HL the horizontal Line, and C the Center of the Picture. In like Manner, the Diftance of the Eye for each Pair of Scenes is to be determined; ;-Thus Ed (Fig. 97) is the Diftance of the Eye from the Scene $f b$; therefore, fet off CE in this Figure equal to Ed in the 97th Figure: And having got the proper Diftance of the Eye for one Pair of Scenes, $\mathcal{F} c$. we are to proceed with our Work in the very fame Manner as if it was an upright Picture. And the fame Methods are to be taken for all the other Side Scenes, the Back Scene, © $\mathrm{E}_{\mathrm{c}}$. taking their Breadths from the Plan, and their Heights from the Elevation: All which may be very eafily done by drawing a fmall Model, aecording to the above Rules, and then transferring the feveral Parts unto each Scene, Ȩc.

Or this may be done by confidering the Curtain as a Picture which is to reprefent the whole Defign, and upon which are drawn the feveral Parts proper for each Scene; then by reticulating the whole, as in the 10 ift Figure, we may transfer the Part peculiar to each Scene, in the fame Manner as one Picture is copied from another by the common Method of Net-Work: But we muft take great Care to divide each Scene exactly in the fame Manner as that Scene is divided by the Reticulation upon the Curtain.

And here it is neceffary alfo to obferve, that fince the Space If, (Fig. 97) which is the Diftance between the Scenes In and $f b$, repiefents the whole Space from I to o; therefore, no Bart of the Diftance Io Should be drawn upon the Scene fb; but all that
comes within that Diftance, fhould be painted upon the Scene In: And $f \circ{ }^{\circ}$ of the reft.

Again, we muft take Care to give each Scene fuch a Projection, that a Line drawn from the Eye through the Edge of one Scene, may cut its fucceeding Scene in a proper Manner; as was obferved before: For which Purpofe we may uie the following Method.-..-
Fig. 99. Set off the feveral Widths for the Opening of the Curtain, and Width of the Scenes, from the 96 th Figure, upon the Line af, (which I here fuppofe the Bottom of the Model;) draw alfo the horizontal Line, $\mathcal{E}^{c} c$. then, from the Points $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}$, draw Lines to C, and make ag equal to In (Fig. 97 ;) then draw gm parallel to af, and fet off the feveral Divifions $\mathrm{gh}, \mathrm{hi}, \mathrm{E}_{\mathrm{c}} \mathrm{C}$. from g towards m ; then draw Lines from all thofe Points to C , as in the Figure. Thus again, fuppofe no the Seat of the firft Scene ; then draw $n \mathrm{p}$, cutting. Ck in p ; and then is np the Height of the firft Scene. Again, from the Point 2, where the Edge of the firft Scene cuts CC , draw 12 , which will cut Cd in I ; then is 12 the apparent Breadth of the fecond Scene: And fo of the reft.---In the 100 th Figure is a Set of Scenes compleated; where C is the Back Scene, which parts in the Middle; $\mathrm{I}, 2,3,4$, the Side Scenes; and the prickt Liues $a b ; \mathcal{O}_{6}$ are the Hanging Scenes.


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XXV.


## CH A P. VII.

An Abfract of Several Methods of PerSpective; transcribed from the moft eminent Authors.

THE older Books which I have met with upon the Subject of Perfective, are, one by Vignola *, and another by MAdolors $\uparrow$. And there two Authors feem to me, to have laid the Plan for every Syftem of Perfective (except Dr. Taylor's and Mr. Hamilton's) fince their Times; though few of the Authors who have built upon their Principles, have been fo generous as to acknowledge their Obligations to them; but, on the contrary, have feet off their Books with pompous Titles, to allure the Public, and to raife in them an Expectation of finding fomething new and curious. This, though a Practice too common among Authors, is, in my Opinion, an unpardonable Tranfgreflion of the Rules of Modefty and Plain dealing; and therefore, to avoid any Imputation of this Kind, I have conftantly acknowledged my Obligations to every Author who has lent me any Affiftance. It was for this Reafon, principally, that I gave my Book the Title of Dr. Brook Taylor's Perspective, sic. But though I mut acknowledge my Work to be generally built upon the Principles of that ingenous Author, I hope, I may at the fame Time affert, that whoever will compare my Schemes with thole that have been before made publick, will find very few but what are intirely of my own Invention.

The following Examples, which are taken from Vignola, Marolois, Vredeman Friese, the Jesuit, and Bozo, will be fufficient to chew how one Author has copied from another, and the various Methods which have been publifhed. I hall begin with Vignola's.

[^38]Fig. 102. Here $A C$ is a perpendicular Section of the Picture, $A B$ is the Bottom of the Picture, and C the Center of the Picture, E the Eye, and ES its Height, D is the Elevation of the Cube, and F its Plan upon the Ground. Now, having fettled the above Requifites, draw Lines from every Corner of the Elevation D, to the Eye E, and from the Plan F draw Lines from every Corner to the Seat of the Eye at $S$; and their feveral Interfections upon the Line BC, will give the proper Meafures for the Height and Depth of the propofed Reprefentation. Thus, from the Points 1, 2, 3, 4, on the Line of Elevation AC, draw Lines parallel to the horizontal Line; then from the Line $A B$ of the Plan, take $A b, b a$, and fet from I to a, and take Ac, cd, and fet from 2 to $d$; which will give the proper Heights and Depths, as in the Figure. Or, by fetting off A 5,56 , equal to 78,89 , and drawing Lines to C, we may get the Depth of the Plan abcd.

By this Method, we are taught how to make a perfpective Scale for any Reprefentation: For having drawn the Elevations and Plans of the propofed Objects, the Line AB may be confidered as a Scale for the Plans, and the Line AC as a Scale for the Elevations.

## II. MAROLOIS's METHOD.

## To put a Double Cross into Perfpective.

Fig. 103. Here ce is the Ground Line, DC the horizontal Line, C the Center of the Picture, and CD the Diftance of the Eye.-Draw out the Plan of the Crofs, as A, and put it into Perfpective, as in the Figure ; then, at any convenient Diftance c, raife a Perpendicular cd upon the Ground Line, and fet the Elevations a, 1, 2, b, upon it ; then from c, 3, 4, d, draw Lines to any Point $H$ in the horizontal Line; after which, draw Lines through every Angle of the Plan, parallel to the horizontal Line, which will cut the Line cH , and thereby give the Points by which the Perfpective B of the Elevation may be compleated; finally, from every Angle of the Plan draw Lines perpendicular to the horizontal Line, and from every Angle of the Elevation draw Lines parallel to the horizontal Line; and then, their mutual Interfections with each other, will produce the propofed Reprefentation, as in the Figure,-The Reader is defired to compare this with my Method in the 40 th Figure.

## Different Methods of Perfpective.

III. JAN VREDEMAN FRIESE's METHOD.* Toput a Cube into Perfpective.
Make the Bottom BP of the Picture a Scale of Feet, from whence Fig. 1040 find the Reprefentation of any Number of Geometrical Squares, as in the Figure- Now let it be required to find the Appearance of a Cube abcd, equal to two Feet in Diameter, and let it be one Foot from the Bottom of the Picture.--Make the Front Face abcd two Squares wide and two Squares high, then give two Squares for the Depth, and from thence compleat the Figure.

## IV. The JESUIT's METHOD. $\downarrow$ To put a CUBE into Perfpective.

Draw the Plan ABCD, which put into Perfpective, as a bcd; Fig. 105: from thence draw another Plan ef $g \mathrm{~h}$, then, by Marolois's Method, find the Elevation, and from thence compleat the Figure. And this fame Method is taught by Kircher, in his Work, entituled, Ars magna Lucis et Umbra, Chap. 3.

## V. ANDREA POZZO's METHODS.\|

## 1. To put a Parallelopiped into Perfpective.

Draw the Elevation A, and from thence the Plan B; then put Fig. rof the Plan into Perfpective, as a gfd ; from the Corner a of the Plan, erect the Perpendicular ab, and continue the Top of the Elevation A 'ti!! it cuts abin c; from whence the Perfpective Elevation may be compleated by Marolois's Method : And having got the Depth of one Plan, and the Height of the Elevations, the whole Reprefentation may be compleated by Vignola's Method.
2. To put a Parallelopiped into Perfpective, which will ex:plain Pozzo's other Metbod.
Here in Conformity to Vignola's Perfpective Scale, AC is the Fig. ros: Section of the Picture, AB the Ground Line, D the Eelevation of an Object, and F, H, the Plans of two Objects parallel to the Pic.

[^39]$$
\text { L2 } \quad \text { ture; }
$$ as the Seat of the Eye.- From the feveral Angles of the Elevation draw Lines to the Eye E, and from the feveral Angles of the Plans draw Lines to I, which will cut BC, and thereby give the Elevations and the Depths of the Plans; from whence the 108th Figure may be compleated. Thus, A $1, A_{2}, A_{3}, A_{4}$, are each equal to their correfponding Divifions, $A_{1}, A_{2}, A_{3}, A_{4}$, upon the Line of Elevation A C, Fig. 107; and I O, It, छc. are equal to $\mathrm{Ko}, \mathrm{Kt}, \mathcal{E} c$. of the fame Figure; and AC is equal to the Height of the Eye SE. But I have put every Line and Point, to explain the Thing the better.

There are feveral other fmaller Treatifes upon Perfpective, and particularly one by Bernard Lamy, entitled, Perfpective made Eafy, which, as it contains fome curious Obfervations upon Painting, $\mathcal{E} c$. is worthy of Notice.

## FINIS.

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## A $\quad$ P $\quad$ P $\quad$ E $\quad N \quad D \quad I \quad$.

THE favourable Recieption of the firt Impreffion of this Work had been a fufficient Inducement for publihing a fecond Edition, if the Number of my Subfcribers had not made it abfolutely neceffary.
When I firf engaged in this Undertaking, I much dreaded the Difficulties which prefented themfelves, both from my own Incapacity, and from the Nature of the Subject: For although I had made Perfpective my particular Study for feveral Years, and was fatiffied in my own private Opinion as to the fhortnefs and clearnefs of the preceeding Method; yet to make it intelligible to others, and ufeful in general, were Things not to be accomplifh'd without much Study, Labour, and Expence. I therefore determined to proceed very cautiounly, to view every Article in various Lights, and not to print any thing without having it firft approved of by competent. Judges.

As this feem'd the mof likely Means to prevent my publifhing any ufelefs or undigetted Figtures, fo I thought it alfo a very likely way of avoiding the little ill-matured Criticifms, which are fo often made: upon the Works of a young Author: And I muft confefs (with the utmoft Gratitude and Thanks) that my Succefs hath abundantly exceeded my utmof Expectations; for I have been fo fortunate as to have the Work approved of in general, and recommended in fuch a peculiar Manner, by Gentlemen of great Genius and Knowledge, that I now begin to think it fecure, from public Cenfure, under their kind and powerful Protection.

But it may be neceffary to inform my Reader of the Additions which. he may expect to find in this Appendix: And; in the firt place, I have more largely and more fully confidered the Perfpective of Shadows; I have alfo given one Figure to thew why a Down-hill (if merely confidered as fuch) cannot be reprefented upon the Picture; then I have: added another Figure, to explain the nature of what is called a Bird's-eye-view, a fort of Perfpective ufed in drawing Fortifications, and the like. I have alfo fhewn the Ufe of an Inftrument of my own Invention,

## A $\quad$ P $\quad$ P $\quad$ E $N$ D $\quad \mathrm{I} \quad \mathrm{X}$.

which may be of Service in Drawing extenfive Views, large Buildings, $\mathcal{E}^{\circ} c$. and, laftly, I have given the Conftruction of a fmall Pocket Camera Obfcura. -To begin therefore with the Additions to the Perfective of Shadows.

In both the Theory and Practice of Shadows, I have frequently made Ufe of this Expreffion, viz. "The vanifhing Point of the Sha" dow ;" which, poffibly may require fome farther Explanation: Becaufe the Shadows of any Objects which are compofed of perpendicular or parallel Planes, will, when put into Perfpective, vanifh into various Points upon the horizontal Line ; and therefore this Article may not feem fo very fignificant, as in fact it is.

By the vanifhing Point then of the Shadow, is meant the vanifhing Point of fuch Shadows only, as are fuppos'd to be caft upon the Ground Plane (or upon a Plane parallel to it) by the perpendicular Edges of Objects. For fince thefe Species of Shadows will always vanifh into the Center of the vanifhing Line of the Plane of Rays, therefore this particular vanifhing Point will be found to be more ufeful than any other; as will appear by the following Examples.

And, as I found it neceffary to make fome confiderable Additions to this Part of Perfpective, fo I have made Choice of fuch Figures as might contain the moft general Rules, and have given fome of the moft curious and difficult Examples which can be propofed : And that they may be the more clearly comprehended, we will range, whats we have farther to advance, under the following Heads, viz.
-I. When the "Shadow is caft upon the Groiund, or upon a Plane parallel to it.
II. When the Shadow is caft upon a per pendicular Plane, III. When the Shadow is caft upon an oblique Plane.

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\text { C } A \quad E \quad \text { I }
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When the Shadow is caft upon the Ground, \&c.-Firft, by a perpendicular Object; fecondly, by a parallel Object; and thirdly, by an inclined Object.

EXAMPLEI.—Wher it is caft by a perpendicular. Object.
Pig. i.

* Here R S is given for the vanifhing Line of the Rays of Light, $\overline{\mathrm{R}}$ for the yanifhing Point of the Rays, and A B for the perpendicular Object, whofe Shadow is fought.
* The Line R S, whether continued or not, will always fignify the vanihing Line of the Plane of Rays, and $R$ the vanifing Point of the Rays of Light.


## A P P E N D I X.

From $A$ and $B$ draw $A S$, and $B R$, cuting each other in $b$; then is Ab the Shadow of $\mathrm{AB} .+$
Example II.—When it is caft by a parallel Object, as the Plane 1234 .
Find the Seats of the four Corners upon the Ground, as a, e, n, f; Fig. 8; then from thofe Seats draw Lines to $S$, and from $1,2,3,4$, draw Lines to R, which will interfect each other, and thereby give the Appearance of the Shadow m.---Now becaufe $S$ and $H$ are the vanifhing Points of the Plane 1234 , therefore the Sides of the Shadow wiil vanifh into thofe Points.

## ExAMPLE III.-When it is caft by an inclined Object.

Let I2 34 be a Pyramid, whofe Shadow is required.-From Fig. 7. n, the Seat of its Apex or Top 3, draw a Line to S, and from 3 draw a Line to $R$, cutting $n \mathrm{Smm}$; then from-I and 2. draw Lines to $m$, which will compleat the Shadow.--And in the fame Manner the Shadow of the inclined Edge AB, of the perpendicular Plane ABD, is to be determined.

## C A S E II.

When the Shadow is caft upon a perpendicular Plane.---Firft, when it is caft by a perpendicular Object ; fecondly, when it is caft by a parallel Object, and thirdly, when it is caft by an inclined Object.

## ExAMPLEI.—When it is caft by a perpendicular Object.

Let $a b$ be an Object perpendicular to the Plane ABCD, and let ${ }_{\text {Fig. }}$. it be required to find the Shadow of the Object a $b$ upon this Plane. ---From a and $b$ draw the Perpendiculars $a f, b g$, and through $f$, where af cuts the Bottom $A B$, draw $\mathrm{C} g$, cutting bg in g ; then

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## A P P E N D I X.

from $g$ draw $g S$ cutting $A B$ in $h$, and from $h$ draw a Line at pleafure, but parallel to a $f$, then from $b$ draw a Line to $R$ cutting $h e$ in e; finally, draw a Line from e to a, which will be a Guide for compleating the Shadow, as in the Figure.

## EXAMPLEII.——When it is caft by a parallel Object.

Fig. $\mathrm{I}:$
Here $A B$ is the Object, and 123 , the Plane upon which the Shadow is caft.---From A draw A S cutting the lower Edge 1 2, in a, and from B draw BR, and from a, draw a g parallel to A B; then will af be the Shadow propofed: And was the Plane I 23 'continued as high as $g$, then would ag be the Shadow of A B..-The fame may be faid of the Shadows a, c, in Figures 2, 3.

## Example III.——When it is caft by ans inclined Object.

Fig. 5. In this Figure I 345 , is an inclined Object, which cafts a Shadow upon the Planes A, C.---Find the Shadow of the perpendicular Plane 12 3, upon the Ground, which will cut the lower Edge of the Plane $A$ in a; continue a till it cuts the horizontal Line in $f$; then is $f$ the vanifhing Point of the Shadow of the Edge I 3, therefore from $R$ draw a Line through $f$, and continue it at pleafure, which will pafs through C, (the vanifhing Point of the inclined Face 3345 ) then from H , the vanifhing Points of the perpendicular Planes A, C, draw HV perpendicular to the horizontal Line, which will cut $R V$ in $V$, and thereby give $V$ for the vanifhing Point of the Shadows $a b$ and $c d$; therefore from a draw $a V$, and from $b$ draw bS , from c draw cV , and finally, from 3 draw 3 R , which will give the Point d for the Shadow of the Corner $3 \ldots-$ And in order to find the vanifhing Point of the Shadow which is caft upon the Plane G, by the Top 34 of the inclined Plane, continue V S below the horizontal Line, and draw a Line from $R$ parallel to the horizontal Line, which will cut V S in $S$, and thereby give $S$ for the vanifhing Point of that Part of the Shadow ; as is evident by the Figure.

## C A S E III.

When the Shadow is caft upon an inclined Plane.---Firft, by an Object perpendicular to the Ground ; and fecondly by an Object inclined to the Ground.

## A P P E N D I X.

EXAMPLE I. When it is caft by a perpendicular Object; wibich will admit of great Variety.
Ift. If the vaniming Point $S$ of the Sides 15,34 be the vanifh- Fig. 8: ing. Point of the Shadow upon the Ground, then will $S$ be alfo the vanifhing Point of the Shadow upon the inclined Plane.-Thus the Shadow cd which is caft by AB upon the inclined Plane will vanifh into $S$.

2dly. If the vanifhing Point $S$ of the Shadow be taken within ${ }_{\text {Fig. }} z_{3}$ the vanifhing Point H of the Edges $\mathrm{I} 5,34$, then the vanifhing Points of the Shadow cd, will be above the horizontal Line; and may always be found in this Manner ; viz, find the Shadows A a, a c, and continue the vanifhing Line RS above the horizontal Line at Pleafure ; then from the vanifhing Points $H, V$, of the inclined Plane draw HV, cutting Rs in $s$, and then is $s$ the vanifhing Point of cd.

3dly. If the vanifhing Point $S$ of the Shadow be taken without Fig. 3: the vanifhing Point H of the Edges 15, 34, then the vanifhing Point $l$ of the Shadow cd will be below the horizontal Line: And this is found by drawing a Line through the vanifhing Points $V$ and H of the Edges 13,15 of the inclined Plane, till it cuts the vanifhing Line R S in 1.

Now the Reafon of all this muft appear extremely evident, if we confider, firft, that however a Shadow is caft upon any Plane, it mult vanifh into a Point or Points in the vanifhing Line of that Plane; becaufe the Boundaries or Out-lines of every Shadow, are confidered only as Lines drawn upon a Plane. And fecondly, becaufe the vanifhing Points of the Sides of any Shadows, and the vanifhing Point of the Plane of Rays which projects thofe Shadows, muft always be in the fame Plane: But, as we obferved before, this will more fully appear by the Figures.
4thly. To find the Shadow of A B CD upon the inclined Plane Fig. 4: I 3 4.---Here VL continued will be the vanifhing Line of the inclined Plane: And to find the Shadow cd, firft determine the Shadow of A B CD upon the Ground, which will cut the lower Edge of the inclined Plane; then continue R S till it cuts VL in V, and from R draw R L parallel to the horizontal Line which will cut VL in L; and then are $L$ and $V$ the vanifhing Points of the Shadow, as in the

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Figure-From this Figure we may obferve, that the Line V L paffes through the vanifhing Point $H$ of the Edges 1, 34, and therefore the vanifhing Point $V$ may be found by drawing $L V$ through H.
Fig. 6. 5 thly. To determine the Shadow of the Pillar A, when it is caft upon two inclined Planes.-Here HF and FL are the vanifhing Lines of the inclined Planes I 24,234 . -Find the Shadow of the Pillar upon the Ground which will cut the lower Edge of the Plane 124 in ab; then continue $H F$ till it cuts $R S$ continued in $V$, and then is $V$ the vanifhing Point of the Shadow abcd, and fo alfo $s$ where the vanifhing Line FL cuts $R V$; is the vanifhing Point of the Shadow cdef.

> ExAMPIE II.—When the Sbadow is caft by an inclined Line $A B$, upon an inclined Plane 123.

Fig: 7. Having found the Shadow of $A B$ upon the Ground, continue it till it cuts the horizontal Line in s; then from R draw a Line thro' $s$ cutting the vanifhing Line $H V$ of the Plane 123 , in $f$, and then is $f$ the vanifhing Point of the Shadow ab ; And if Lf be continued it will cut the vanifhing Line $N \mathrm{U}$ in U .
Fig. 8. To find the Shadow of a perpendicular Object when it is caft upon a Tetrahedron.-Determine the Shadow A b of AB upon the Ground, and draw the Seat a enf of the Plane I. 234 ; then from where the Shadow cuts a $f$, draw Lines parallel to $A B$ cutting the Edge I 4, which will be a Guide for drawing the Shadow upon the upper Face, to $S$. And for the Shadow on the Face 14 D , continue the vanifhing Line HM of $1{ }_{4} \mathrm{D}$, and SR the vanifhir Line of the Plane of Rays, and their Interfection with each other will be the vanifhing Point of that Shadow.-As to the Shadows which are caft upon the Ground by the above Objects, it is prefumed, that they want no farther Explanation.

Thefe are a few of the many Examples which might be produced as a farther Illuftration of the Perfective of Shadows; for this Part of Perfpective might be extended to Infinity: However thefe Figures contain fome of the moft general Principles that I can think of, and are abundantly fufficient to fhew how the Appearances of any Shadows are to be exactly determined, upon all forts of Planes, and in the moft dificult Situations.

Some

## A P P E N D I X.

Some Confiderations upon drawing the Reprefentation of an inclined Plane going from the Eye, or wobat is ufually called a Down-bill.

To reprefent a Down-hill hath always appeared a Matter of great Difficulty to Painters, and this will ever remain impracticable, fince, in the Nature of the Thing, it is impoffible to be done.

For let $\mathrm{HL}_{2}$ be the horizontal Line, and let $\mathrm{F}_{1}, \mathrm{~F}_{2}, \mathrm{~F}_{3}, \mathrm{~F}_{4}$, Fig. 9. and FH, reprefent the feveral Angles, or Inclinations of five different Hills: Then we may conceive thefe Hills to be like fo many inclined Planes. And if they are fuppofed to vanifh into Lines parallel to the horizontal Line, thien a a is the vanifhing Line of the Hill. $\mathrm{F}_{2}$, b b of the Hill F 3, cc of the Hill F4, d d of the Hill F H, and the horizontal Line HL is the vanifhing Line of the even Ground FK; for the feveral Lines LC, Ld, $\mathcal{E} c$. are parallel to the original Lines F K, FH, $\mho^{\circ}$. From whence we may obferve that the even Ground (fuppofe a Roadl A B C) will feem to rife upwards, and vanifh into the horizontal Line HL; and the leaft inclined Plane will vanifh below the horivontal Line, but will take up the Space D d upon the Picture ; the next inclined Plane will take up the Space Dc, the next Db , and the next: Da ; and when the Inclination of the Hill is fo great (as F I) that its vanifhing Line will fall below the Bottom of the Picture, then that inclined Plane will totally difappear, and therefore can have no Place upon the Picture. So that from: hence appears the Impoffibility of reprefenting a Down-hill, (fingly as fuch) by the Rules of Perfpective; becaufe, what we actually know to go down-hill in Nature, will, if ever fo correctly drawn upon the Picture, appear to rife upwards; which is another ftrong. Inftance of the Infufficiency of Perfpective upon fome particular Occafions. For in orderr to reprefent fuch an inclined Plane, we muft have Recourfe to Experienee, which will teach us to difpofe particular Kinds of Objects in fuch a Manner, as fhall convey to the Mind the Idea of a Down-hill; fuch as fhewing Part of a Figure, or making the Tops of lofty Objects full below the horizontal Line, $\mathcal{E}_{c} c . \mathcal{E}_{c}$ c._As to the Manner of reprefenting Hills when they are fideways with the Picture, that is fo very eafy, as not to be worthy Notice in this Place.

## A $\quad \mathrm{P} \quad \mathrm{P} \quad \mathrm{E} \quad \mathrm{N} \quad \mathrm{D} \quad \mathrm{I}$ X.

Of a Bird's Eye View; and bow to put a Fortification, Ecc. into Perfpective.
Although this Part of Perfpective is eafily deducible from our general Rules, yet I have here added the following Figure, which is fufficient to explain the whole of this Matter. And I have made the Figure very fimple, with upright Walls only, and without Baftions, or any the leaft common Parts of Fortifications.

In drawing the Reprefentations of Fortifications, it is neceffary not only to fhew one View as feen upon the Ground, but to exhibit alfo fo much of the feveral Buildings as the Eye can poffibly take in at one Time from any Situation. And in order to do this we muft fuppofe the Eye to be removed to a confiderable Height above the Ground, and to be placed as it were in the Air, fo as to look down into the Building, like a Bird that is flying.
Fig. 10.
Suppofe therefore M, N, O, to be the Walls of three Fortifications, the loweft $(\mathrm{O})$ of which, is furrounded with a Ditch.- Now to draw thefe feveral Reprefentations, we muft firf choofe a proper Height for the horizontal Line, and then proceed exactly in the fame Manner, as if we were drawing any Objects by the common Rules; only obferving to let the Diftance we work with, be fomewhat greater than the Space between the Bottom of the Picture and the horizontal Line. And, if it were required, to draw the Appearance of a Ditch, or the like; then from the Surface of the Ground, as IK, fet off I 2 equal to the fuppofed Depth, and draw a Line to the vanifhing Point of the top Edge of the Ditch; and fo ${ }^{2}$ kewife for the Surface of the Water, which we will fuppofe to be at the Diftance I I from the Top of the Ditch. Set this Diftance from I to I , which will be a fufficient Guide for the above Purpofes.

It is eafy to conceive that the higher the horizontal Line is placed, the more of the Fortification will be feen, and the contrary the lower it is placed.

## A Defcription of an Inflrument that may be ufeful in taking extenJive Vierws, ङ゚゙C.

The Ruler A B is 19 Inches long, and is graduated into 19 equal Parts; upon the upper Edge of it is a douftail Groove to receive the perpendicular Ruler G, which has one End fitted to it, fo as to flide very eafily; this Ruler is 15 Inches long, and is divided into 15 equal

## A P P E N D I X.

Parts, and upon the Back-fide of it (reprefented by F) is a Line drawn exactly in the Middle, and upon this Line is fixed a piece of Barber's Silk, with a little Plummet at the End. The Ruler A B is fixed by two Screws a c, to two pieces of thin Brafs; and thefe pieces of Brafs are fixed at the other Ends by two Screws de to a ftronger piece of Brafs bf ; this Brafs b f goes clofe to the Ruler A B, and has a Joint at $x$ which turns upon a Screw; below this Joint is a piece of round Brafs about fix Inches long, which goes into a Hole made in the Top of the Staff, and may be raifed higher or lower like a Barber's Block by means of the Screw f; Part of this Staff is CDE, and the whole Length is about 3 Feet, and at the Bottom is, what we call a rank Screw made of Iron and is fixed to the Staff. H I is a Wire 22 Inches long, with a Screw at h to go into the Hole b; the piece of brafs Wire, bent into the Form ik is fixed to the Wire H I by the Screw k; and the Part i goes into the Hole f, in the brafs Piece bf. The fmall Wire KL. is about 12 Inches long and is flatted at K , at which Place is a little Hole about $\mathrm{I}-8$ th of an Inch in Diameter; this Wire K L is fitted to the Holes $1, m, n, o$, which are made in the larger Wire H I, and it may be placed higher or lower, by means of a fmall Screw.-This is a Defcription of the feveral Parts of the Inftrument; we will next fhew its Ufe.

Fix a Paper upon a Drawing-board, as in Fig. 12, and divide the Paper length-ways into 19 equal Parts, and Perpendicularly into 15 equal Parts; and in Proportion as you intend the Drawing to be larger or fmaller, make thefe Divifions greater or lefs. Then take the Staff and fix it ftrongly in the Ground, by means of the Screw at the Bottom, and at a convenient Diftrance from the Profpect which you intend to take. After this, put the Inftrument together as in Fig. 13; and fix the Ruler A B, exactly Horizontal by means of the Plummet on the perpendicular Ruler and the Brafs Joint x; then fix the Wire K L. fo as to have the Eye-hole exactly level with the Horizon, that is equal to the Height of the Eye, and take care alfo to have the greateft Diftance of the Eye-hole from the Ruler, equal to the whole Length of the longeft Ruler A B, and never lefs than the Diftance h 1.-Having thus fixed the Inftrument, place yourfelf on a Seat, and proceed to make your Drawing in the following Manner.Look through the Eye-hole, and then move the perpendicular Ruler in the Groove, till you get one E.dge exactly againft fome principal Object ; then will the Parts upon the Ruler fhew how high the Object is from the Bottom of the Ruler (that is from the Bottom of the

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Picture) and you will alfo have its apparent Height ; therefore tranffer this unto the Paper in thefe Squares which correfpond with the Divifions upon the Rulers. So alfo for the Breadths of Objects ; move the perpendicular Ruler fo as to be even with the Sides of an Object ; and the Divifions upon the lower Ruler will fhew their apparent Breadths. And after the fame manner, get the Places and apparent Sizes of as many principal Objects as are neceffary for affifting you in compleating the whole Drawing; which may be done by this Method with great Exactnefs. - And having finifhed the Drawing, the Inftrument may be taken to Pieces and put into a Box, which may ferve as a Drawing-board; the Top $M$ may be feresved into the staff which will ferve as a Walking-ftick, and the Stool to fet on may be made very Portable, fo that every Part of this Apparatus may be carried by one Perfon with great Eafe. *

I fhall juft Obferve that the Inftruments which have been Publifhed of this Kind, have no Diftance limited for the Eye hole, which make all the Reprefentations that are drawn by an improper Diftance moft egregioufly Falfe; as is Demonftrated in what we have faid concerning the Diftance of the Eye in Chap. 6. B, I. and Chap. 2, B, 2.

The 14th Figure is a fmall pocket Camera Obfcura. The lower Part of this Inftrument is a fquare Box, 4 Inches in Diameter with a Looking-glafs E fixed at an Angle of 45 Degrees. In the Middle of the Side BC is a fmall Hole 2 Inches in Diameter, in which goes a Tube to Slide 2 Inches long, and in that a Lens for the ObjectGlafs. The Top part F of this Box is a Piece of ground Glafs to receive the Image from the Looking-glafs E. But as the Picture will be very fmall, and confequently the Object's too much diminifhed; therefore, on the Top of the Box CD a b, is another Tube G, with a Lens of a large magnifying Power; which being raifed higher or lower will fo increafe the Size of the Picture as to make the whole View very diftinct. The Fore-part which is left open, may

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## A $\quad$ P $\quad$ P $\quad \mathrm{E} \quad \mathrm{N} \quad \mathrm{D} \quad \mathrm{I}$ X.

be either made like two Doors to move upon Hinges, or may flide in Grooves for that Purpofe: One of which is abfolutely neceffary on account of cleaning the Glaffes \&cc. - This with the other Inftrument may be had of the fame Perfon, mentioned in Page I Book I and of Mr. Adams in Fleet-Street.

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[^0]:    * Mathematical Projection comprehends all kinds of Projection whatoever ; fuch as the Projection of the Sphere, the Cylinder and its Sections, Conic Sections and the like.

[^1]:    - Vid. Leonardo da Vinci upon Painting, p. 31.
    + Ibid. p. 29. Frefnoy's Poem upon Panting, p. 19, v. 315. And Du Piles mpon Painting, Chap.18.

[^2]:    * Mr. John Bennet, at the Globe in Crown Court, between St. Ann's and Golden Square, London, has had particular Directions from the Author, for making a very fimple and useful Cafe of Inftruments, fit for the above Purpofe.

    Book I.
    A.

    Point;

[^3]:    For right Angle, fec Geometrical Definitions in the next Seation.

[^4]:    * Vide Smith's Opt. P. 17, Art. 57, 58.
    + The Axis of the Eye is a Line drawn through the Middle of the Pupil and of the Cry ${ }^{6}$ falline Humour, and confequently falls upon the Middle of the Retina. And the Axes of tboth Eyes produced, are called the Optick Axes; which will be better underfood after the Defcription of the Eye.

[^5]:    * By a Lens in this Place is meant a Glafs which collects the Rays of Light into a Point, like a common Burning-Glafs.
    + When a Ray of Light paffes out of one Medium into another of a different Denfity, it will be bent near the Surfaces of thofe Mediams, which bending is called Refraction.
    $\pm$ If feveral Rays approach each other fo as to meet in a Point, they are faid to converge; and if they proceed from a Point and go further off continually, they are then faid to diverge.
    \# Vide Smith's Opt. p. 26.
    \& Vide Newton's Opt. p. 11.

[^6]:    * A Perfon may eaflly fatisfy himfelf of the Truth of this, by only taking a common Burn-ing-Glafs in one Hand, and a Piece of white Paper in the other, and let him hold the Glafs before any Object, and the Paper on the oppofite Side of the Glafs; then by moving the Glafs or Paper backwards and forwards till he gets the Rays to their proper Focus, he will fee the
    

[^7]:    - Vide Newton's Opt. F. 12.
    it Vide Clarke's Rohault, vol. 1. p. 249.
    $\ddagger$ Ibid,
    II 'The Rays of Light OA and OB, which come from the extreme Points of the Object to the Eye O, form an Angle AOB, which is called the Optick or Vifual Angle; and the 2ays $O A$ and $O B$ are called Vifual Rays.

[^8]:    * Vide Smith's Opt. p. 29.
    - By apparent Magnitude is here meant the Bignefs of the Picture upon the Retina.

[^9]:    - Viale Smith's Opt. p. 7.

[^10]:    * The Truth of all this any Perfon may convince himfelf of by making the Experiment, or by only holding a Prifm between his Eyes and the Sun; then by turning it round, he will fee the feveral Colours in their proper Order, as above defcribed.
    $\dagger$ Martin's Philof. vel. II. p. ${ }^{5} 56$.-Hamilton's'Perf. p. $x_{2}$

[^11]:    See Definition 6, Sect. II. of this Chapter.

[^12]:    - Vide Page 25.
    + The original Plane OGLP, which is perpendicular to the Picture, I fhall always fupofe the Ground, unlefs mention be made to the contrary; becaufe it will be more intelligible to the Generality of Readers, and becaufe I fhall malke great afe of this Plane, and of its vanifhing Line $H L$, as being the Horizontal Line.

[^13]:    * See Definition 2, Sea. 2. Chap. 1.-alro Definitions 4, 5, Seat. 2, of this Chapter.

[^14]:    - To Reticulate any Surface, is to divide it into Squares like Net-Work

[^15]:    * What is here faid to be given, is exclufive of the Diftance of the Eye, the Center of the Piture, Goc $^{\circ}$. which, it is prefumed, will be taken for granted, withoat mentioning them.

[^16]:    * The ingenious Dr. Smith, in his Treatife upon Opticks, has given us feveral Inftances of the Fallacies in Vifion, amongtt which, he fays, "We are frequently deceived in our Effi"t mates of Diftance by any extraordinary Magnitude of Objects feen at the End of it : As in " travelling towards a large City or a Caftle, or a Cathedral Church, or a Mountain larger "than ordinary, we think they are much nearer than we find them to be upon Trial. For
    " fince by Experience the Ideas of certain Quantities of known Diftanees are ufually annexed
    " to the appasent Magnitudes of known Objects of a common Size; atd fince the apparent
    "Magnitudes of thofe larger Objects at a greater Diftance are the fame as of the fmaller at
    " a fmaller Diftance, it is no Wonder they fuggeft the ufual Idea of fraller Diftance annext
    " to more common Objects. This is further evident, becaufe we are ignorant of the Coun-
    "try, and of the Inequalities in the Ground interpofed." Again, he obierves, "the Part of
    " the Monument extant above the Tops of the adjoining Houfes, I am told, is five times
    " longer than the Height of the Houfes, and yet from below that Part appears but two or
    "t three times longer at moft ; becaufe of its unufual Magnitude and Obliquity to the Sight."
    And the fame curious Gentleman adds, "I remember a red Coat of Arms, upon the Top of
    " an Iron Gate at the End of a Walk, was taken for a Brick Houfe in the Fields beyond
    " it". Vide Smitb's Opticks, Book I. p. 61, 62.
    † In regard to Perception, that acute and judicious Reafoner Mr. Locke, obferves, "We
    ". are to confider concerning Perception, that the Ideas we receive by Senfation are often in
    " grown People alter'd by the Judgraent, without our taking Notice of it. When we fet
    " before our Eyes a round Globe, of any uniform Colour, v.g. Gold, Alabafter, or Jet, 'tis
    " certain, that the Idea thereby imprinted in our Mind, is of a flat Circle variounly fhadow'd,
    " with feveral Degrees of Light and Brightnefs coming to our Eyes. But we having by ufe
    " been accultom'd to perceive, what kind of Appearances convex Bodies are wont to make in
    " us; what Alterations are made in the Reflections of Light, by the Difference of the fenfible
    "Frgures of Bodies, the Judgment prefently, by an habitual Cuftom, alters the Appearances
    " into their Caufes: So that from that, which truly is Variety of Shadow or Colour, collect-
    "* ing the Figure, it makes it pafs for a Mark of Figure, and frames to itfelf the Perception

[^17]:    * Of this we have feveral Inftances in Pozzo's firf Book upon Perfpective, particularly in Fig. 45, 46, 50, and $5^{1}$; in which, thofe Columns that are fartheft from the Point of Sight, are fo prodigioufly increafed in their apparent Widths, as to lofe very near one Diameter in Height; and, I think, the Difproportion is too vifible to be difputed, efpecially in the $45^{\text {th }}$ and 46 th Figures.

[^18]:    *. But it is the Opinior of fome very eminent Painters, that the Words Chiara Ofruro, more properly fignify a Clearnefs of Shadow.

    Parts,

[^19]:    * The Infrument upon which a Pieture is placed to be painted, is called an Eafel; and therefore, every Pitture which is moveable, is called an Eafel.Piece.

[^20]:    - An Equilateral Triangle is that whofe Sides and Angles are all equal.

[^21]:    * This Angle may be fet off with an Inftrument called a Protractor, which is a Semig Circle divided into 180 equal Parts, called Degrees.
    + The Learner cannot make this Figure 500 familiar too him , as 'tis of prodigious Ufe.

[^22]:    * A right Angle is one Corner of a Square, or 90 Degrees.
    + Bifeet, is to divide any thing into two equal Parts: Thus, on $E$ defcribe any Are, $O Q P$, then divide that Arc, in Q, into two equal Parts, and draw E B , and then is the
    Angle KEL bifected.

[^23]:    * A Hexagon is a fix-fided Figure ; and when its Sides are all equal, it is called a regular Hexagon.

[^24]:    * I mean fuch Forms only as are proper Subjects for Perfpective; for as to Objects which. are compofed of an infinite Variety of Curve-Lines, I will not pretend to give any Rules for determining their Appearances according to the Arict Rules of this Art; and was I able to do it, I hould think it unneceflary; fince a good Eye, in fuch Cafes, will direct the Hand with more Eafe, if not with as much Certainty, as any Rules whatfoever; efpecially if the Perfon has a general Notion of Perfpective.

[^25]:    * What I here mean by a Scale of Feet, is not to make a Scale of fo many Feet long, but only to divide the Bottom of the Picture into fuch a Number of equal rarts, which are to be confidered as fo many Feet: A Thing very common amongft Workmen.

[^26]:    * Suppore $F D$ is equal to $F A$, then $c b$ mutt be made equal to ea,

[^27]:    Suppofe A B is four Parts, and BR one Part; then divide DIL inte five Parts; and then So HE to ED as RB to BA; that is, as one to fourt

[^28]:    * Regular Solids, are Bodies terminated by regular Planes, and are five in Number, viz. 1. the Tetrahedron; 2. the Hexahedron, or Cube; 3. the Octahedron; 4. the Dodecahedron; and 5. the Icofahedren: The firf of which is compofed of four equal and equilateral Triangles; the fecond, of fix geometrical Squares; the third, of eight equal and equilateral Triangles; the fousth, of 12 regular Pentagoss; and the fifth, of 20 equal and equilateral
    Triangles.

[^29]:    * Thefe two Examples are fufficient to point out the Method for determining the RepreSentation of all the regular Solids; for having got the Inclinations of their feveral Planes, Efs. their seculiar vanifhing Lines and Points may be found with the greaten Eafe.
    $t$ A Canted Cube, is a folid Body comprehended under is Geometrical Squares and eight equal and equilateral Triangl.s.

[^30]:    - Thefe Methods are applied to practice in feveral of the preceding Figures.

[^31]:    * In this Cafe the Projection of the Capital, (according to Mr. Gibbs, from whofe Book I have taken my Proportions) is one fixth Part of its Length, and the Projections of any other Mouldings may be determined in the fame eafy Manner by a Scale and Compaffes.

[^32]:    * This Method for finding the Reprefentation of a Dome upon a flat Cieling, is principally taken from Andrea Pozzo's Firft Book upon Perfpective, publihed by Mr. Jobn Sturt, Engraver, in 1707; and therefore, if what I am going to advance upon the Subject Mhould appear not to be fufficiently clear, the Reader is referred to the above Book.

[^33]:    *See the Additions apon this Head in the Appendix, p. 8 .

[^34]:    - Here the Reader is referred again to the Additions upon Shadowa in the Appendix.

[^35]:    "A Shepherd's Boy (he feeks no better Name)
    " Led forth his Flocks along the filver Tbame,
    "Where dancing Sun-Beams on the Waters play'd,
    «And verdant Alders form"d a quiv'ring Shade."

[^36]:    * Antrea Pozzo, in both his Books upon Perfpective, has alfo been very copious upon the fame Subject.

[^37]:    * The Reafon why Eh is taken for the Height of the Eye, and not EH, is, becaufe Fig. 95: ef $Q$ is confidered as the Ground Plane upon which the Picture is fuppofed to itand.

[^38]:    * Vignola was a famous Italian Architect, who flourished in the Beginning of the 15 th Centory : He wrote a Treatife upon Perfective, which was publifhed in 1544 , by Frlippo de Rolf, with Annotations by Ignatius Danti. It was printed in Folio at Rome, and is in the Italian Language
    + I his Work was printed in Folio at the Hague, is in Latin, and was engraved and pub. limed by Henry Hondius in $16: 5$; and though tedious in its Operations, is nevertheless a very curious Performance.

[^39]:    * This Book is a Folio, in French, was printed at the Hague in 1619. It was corrected by Marolois, and engrav'd by Henry Hondius.
    + This Book is in Quarto, was wrote originally in French by a Jefuit at Paris, was tranAlated into Englifh by E. Cbambers, and was printed at London in 1726 .
    $\|$ His firf Book was publifhed in Latin and Engli/b by Fobn Sturt, Engraver, in 1707; and his fecond Book was publifhed by himfelf, in Latin, in 1700 ; and are both in Folio.

[^40]:    + Now that $S$, the Center of the vanifhing Line R S, is likewife the vanifhing Point of the Shadow of A B, may be thus demonftrated. The Lines A B and RS being parallel, therefore the Plane ABRS will pafs through them both; and fince the Shadow of AB is caft upon the Ground, it muft vanifh into the horizontal Line: And becaufe R S is the vanifhing Line of the Plane of Rays which projects the Shadow, the Point $\mathrm{S}_{\text {mut }}$ mikewife be in that Line, and therefore S, the common Section of the vanifhing Line RS, with LS the vanifhing Line of the Plane upon which the Shadow is caft, muft be the vanifhing Point of the Shadow upon that Plane.-And in the fame Manner all the foregoing Figures upon this Head may be demonftrated.

[^41]:    * But although this Iriftrument is very Simple, and ufeful to any Perfon who is tollerably \{killed in Drawing; yet I mult be fo ingenuous as to acquaint my Readers, that there is another kind of Inftrument, made by Mr. Adams, at Tycloo Brabe's Head, the corner of RaquetCourt, in Fleet Striet London: which is fo conftructed, that any Perfon, without either the knowledge of Perfpective, or the leaft Notion of Drawing, may take the Perfpective View of any Building, Profpect, or Figure, with the greatelt Accuracy. This Inftrument is newly invented, (but upon a Plan of Sr. Cbrifopher Wren`s) by a Clergyinan in the Country, whofe Name I am not Commiffioned to mention; however in Juftice to fo ingenious and ufeful a Production, I have taken the Liberty of Recommending it in this Place.

