THE

PRINCIPLES

OF

PERSPECTIVE

EXPLAINED IN

A GENUINE THEORY;

AND APPLIED IN

An EXTENSIVE PRACTICE.

With the CONSTRUCTION and USES of all ſuch INSTRUMENTS as are ſubſervient to the PURPOSES of this SCIENCE.

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PREFACE.

NO Mathematical Science requires a THEORY more than PERSPECTIVE, and perhaps no one has been less attended with it in Publications of this Kind. The Manner in which some Authors have treated the Theoretical Part has not been satisfactory to me; and therefore, as many others may be of my Way of thinking, I have presented them with a THEORY of PERSPECTIVE which appears to me the most genuine, natural, and perspicuous that possibly can be, as it consists of the fewest, and those the most simple Principles, *viz.* the Motion of a Point in a given Direction, and of a visual Ray, proceeding from it to the Eye at a given Distance, which Ray by its Intersection with the perspective Plane, marks out the perspective Appearance of the *Point or* Object upon it.

The Application of this Theory to Practice, in the common Way, is so easy and natural, that I think what is here said may suffice for that Purpose.

But as there are many important Particulars relative to the THEORY and PRACTICE of PERSPECTIVE not touched upon in Treatises of this Sort, I have added as much on that Head as I judged was necessary, to give the Student of the *Polite* ARTS as just a Notion of this Science as it deserves, and as he ought to conceive of it.

I have purposely avoided all Kind of Prolixity, and studied one Point only here, as in all my other Writings, *viz.* To make the Way to Science as easy and as concise as possible, and with the least Expence of Time and Money. The GREEK *Proverb* asserts, *A great Book is a great Evil*; the LATIN *Adage* says, *Verbum sat Sapienti*; and an ENGLISH *Genius* can *see Day at a little Hole.*

To conclude, this Work has been only the Recreation of a few leisure Intervals from Business, which I did not know better how to improve, in Regard to my own Amusement, and the Emolument of the Public; to which great Object, the Endeavours of every Individual of Society ought to be directed.

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CHAPTER I.

The Genuine THEORY of PERSPECTIVE demonstrated from its *First* PRINCIPLES.

PERSPECTIVE is the Art of delineating upon a Plain Surface the Appearance of Objects, such as they have upon a Glass Plane interposed between them and the Eye, at a given Distance from that Plane.

For since the Sensation or Vision of any Point, is caused by a *Ray of Light* proceeding from that Point to the Optic Nerve (or *Retina)* in the Eye, and as this Ray passes through the transparent Plane in its Way to the Eye, the Point in which it transits the Plane is that which we call the *Place* or *Seat* of that Point upon the *Perspective Plane.*

But this Doctrine of the *perspective Seat* of a *Point*, a *Line*, a *Superficies*, and a *Solid*, must be illustrated by proper Diagrams, that it may become easy and perspicuous; and that Method can only convey *clear Ideas* that is in itſelf *natural*, *ſimple*, and conſequently *genuine*. And ſuch an one we ſhall here endeavour to purſue.

This Method conſiſts in the Conſtruction of a *Perſpective* PRISM, and its Interſection by the *perſpective* PLANE. The *Priſm* is repreſented by YGHIKLMN; and the *Perſpective Plane* by ABCD. The *Interſection* of the *Priſm* and *Plane* is the Triangular Plane O*y*Q, *(See* FIG. I.)

The Baſe of the Priſm IGMK is *the Ground Plane*, in which all Objects in the *Plane of the Horizon* are conſidered, and their Perſpective Seats determined in the Plane AC. The Vertex Y of the Priſm is the Place or Poſition of the Eye at its given Diſtance Y*y* from the Perſpective Plane, which is here ſuppoſed to be poſited directly before the Eye, or at right Angles to the Priſm.

The Part of the Plane AD which ſtands on the Ground, is called the *Ground-Line*; the Point *(y)* is called the *Point of Sight*; and the Line E*y*F (parallel to AD) the *Horizontal Line*, for Reaſons which will appear by-and-by.

Then in Order to determine the perſpective Appearance of Objects upon the Ground or Horizontal Plane IGMK indefinitely continued out towards *(c d)*, we muſt firſt conceive a Point to begin to move from the Plane AC along the indefinite right Line O*d*, which is perpendicular to the Ground Line AD; this Point is at the ſame Time in the Side IN of the Priſm, and drawing the dotted Line YO, that will repreſent the *Viſual Ray*, by which the ſaid Point is ſeen by the Eye at Y.

Let

Let the Point in its Motion along the Line O *d*, arrive at the Situation T, and draw the Viſual Ray T Y by which it there appears. Then it is ſelf-evident that the Viſual Rays YO, YT, are in the Plane or Side of the Priſm IN; alſo that the Ray YT paſſes through the perſpective Plane in the Point *(t)*, which therefore is the *perſpective Seat* of the Point T.

As the Motion of the Point is continued, the Angle TYN conſtantly decreaſes; till at length the Point gets to an indefinite Diſtance, and then that Angle vaniſhes, and the Ray YT coincides with the Line YN, and the Object, now in the *Horizon*, has its perſpective Seat on the Plane AC in *(y.)*

By this indefinite Motion from the perſpective Plane in O to the Horizon, the Viſual Ray YT keeps conſtantly moving in the Side or Plane of the Priſm IN, and of Courſe it muſt trace out by its Paſſage through the perſpective Plane the Line O*y*, which is the common Interſection of the two Planes IN and AC; and therefore the ſaid Line O *y* is the *perſpective Seat* or Appearance of the Line or Side O *d* of the Ground Plane indefinitely extended.

In like Manner if a Point moves from Q along the Line Q*c* indefinitely extended, the viſual Ray will deſcribe the Line Q *y* on the perſpective Plane, in which it interſects the Side GN of the Priſm: Alſo another Point moving from P in the Line P *e* to an infinite Diſtance, the Ray proceeding from it will deſcribe upon the Plane AC the Line P*y*, which is the common Section of that Plane with the vertical Plane of the Priſm HN.

Hence

Hence it is again ſelf-evident, that if a Line QO were to move from the Ground Line to an indefinite Diſtance, in a Direction O *d*, and always parallel to itſelf, or to the Ground Line, it would appear on the perſpective Plane to deſcribe the Triangle O *y* Q, which therefore will be the Perſpective of that indefinitely extended horizontal Plane *d* O Q *c*, deſcribed by ſuch a Motion.

Hence alſo it is manifeſt, that any Line QO, however large at the perſpective Plane, yet removed to an indefinite Diſtance, *viz.* to the *Horizon*, will there ſubtend no ſenſible Angle, but vaniſh in a Point at *(y)*. Therefore all Objects in the Horizon will appear in the Line EF paſſing through the Point *(y)*, which is therefore called the *horizontal Line.*

All indefinite right Lines perpendicular to the Ground Line of the Plane, as O *d*, P *e*, Q *c*, have their *Perſpective* expreſſed by right Lines converging to the Point of Sight *(y)*, as O *y*, P *y*, Q *y*, which are called *Radial Lines.*

The *Perſpective* of any determinate Space QOTR, is thus found on the Plane; *t* and *r* being the *Seats* of the Points T and R on the Ground Plane, O *t* and Q *r* will be the Perſpectives of the Sides OT and QR; and drawing *r t*, that muſt be the perſpective of the Side RT; and ſo the *Trapezium* QO *t r* will be the perſpective of the propoſed Area OQRT.

If OT be equal to QR, then is O *t* equal to Q *r*, and *t r* is parallel to O Q. If OT = QO, then is the Area OTRQ a Square, and O *t r* Q the Perſpective of that Square.

Let

Let the Diagonal OR of the ſaid Square be continued out indefinitely towards *f*; then upon the horizontal Line EF, take *y* X and *y* Z equal to *y* Y, the Diſtance of the Eye from the perſpective Plane; and draw XY. Alſo make O *b* = OI (= Y *y*,) and draw *b a* parallel to OQ or RT; and O *b a* being a right Angle, O *a* will be the Diagonal of a Square, and equal and parallel to YX.

Then if a Point be now conceived to move along this Diagonal Line O *f*, to an indefinite Diſtance, the *Viſual Ray* will contain an Angle with the Line Y X that will conſtantly decreaſe, and at laſt vaniſh, when the Point is in the Horizon; and at laſt, by a Coincidence of the Ray with the Line YX, the ſaid Point in the Horizon will have its perſpective Seat at X in the horizontal Line EF.

During this Motion of the Point in the Diagonal of a Square, the Viſual Ray will trace out the Line OX in the perſpective Plane. Wherefore, becauſe this Line O *f*, and all others parallel to it, converge to the Point of Diſtance X upon the Plane, they are called *Diagonal Lines*.

Hence any indefinite Line drawn from the Point O, between OT and OR, will have its perſpective Seat in ſome Part between O *y* and OX, and every ſuch Line between OR and OQ will have its Seat in the ſame horizontal Line beyond X; and ſuch Points in the Line EF to which theſe Lines tend are called *accidental Points*.

Hence the Line X *y* between the Point of Sight *y*, and the Point of Diſtance X, is the Perſpective of 45 Degrees

Degrees in the Horizon; or the Double thereof X Z is the Perſpective of 90 Degrees in the Horizon.

Hence alſo it is evident that ſince the Original Lines Q *c* and O *f* croſs each other in R, therefore the Seat of the Point R will, in the perſpective Plane, be at *(r)* where the Radial Q *y* is interſected by the Diagonal OX. And from thence the following general Rule is derived for finding the perſpective Seat of any given Point or Object upon the Ground Plane, *viz.*

GENERAL RULE.

From the given Point, as T, *draw a Perpendicular to the Ground Line as* T O; *from the Point* O *draw the radial* O *y*; *and from the ſame Point* O *ſet off the Diſtance* OT *from* O *to* Q *in the Ground Line, and draw the Diagonal* QZ, *and where it croſſes the Radial, as at (t,) there will be the Seat of the Point* T *required.*

CHAP. II.

A DEMONSTRATION of the foregoing general RULE of *Practical* PERSPECTIVE, in regard both to *direct* and *oblique* VIEWS.

THE THEORY of PERSPECTIVE required a Position of the *perspective Plane* ABCD (FIG. 1.) with an Elevation above the *Ground Plane*, which in that Figure (as in most real Cases) is at right Angles to it. But the practical Rule we have thence deduced, admits a *Delineation* in *Perspective* of any Sort of Objects with the utmost Truth and Facility by placing the *Perspective Plane* upon the *Ground* or *Horizontal Plane*.

Thus (FIG. 2.) let ABCD be the transparent or perspective Plane in an horizontal Position, and let T be the given Point whose perspective Seat is to be found on the Plane A C in this Position; and then by the Rule the Process is the same here, as before in the other Figure. For drawing the Perpendicular T O to the Ground-Line A B, and from O, the Radial O y to the *Point of Sight* (y); and then making QO = OT, and drawing the Diagonal QZ to the Point of Distance

tance Z it will interſect the Radial O*y* in the Point *(T)* the *perſpective Seat* of the Point T as required. Therefore alſo O*T* will be the Perſpective of the Line OT.

To demonſtrate the Truth of this Method, it will be neceſſary to make the following Conſtruction in the Diagrams, viz. Take OV in the Ground Line (in both Figures) equal to the Diſtance of the Eye OI (= *y*Z,) and draw ZV, which will be parallel to the Radial YO. Then (in FIG. 1.) there are two ſimilar Triangles formed, *viz.* ITY, OT*t*; and VQZ, OQ*T* (in FIG. 2.)

From the firſt we have this Analogy, IT : OT : : IY : O*t*; and from the ſecond, this, QV : QO : : ZV : O*T*. Now the three firſt Terms in each Analogy are reſpectively equal to each other, that is IT = QV, OT = OQ, and IY (= O*y*) = ZV. Therefore the fourth Term O*t is the ſame in the firſt* with O*T* in the ſecond; conſequently the Interſection of the Radial O*y* and Diagonal QZ is the Seat of the Point T as true by the *practical Method*, in FIG. 2. as by the THEORY in FIG. 1.

If therefore OTRQ be the Square before-mentioned, its Perſpective will be O*Tr*Q as determined by the Theory, and OR and QT will be ſeated in O*r* and Q*T*; the Center N of the Square in *(n)*, and the Sides QR and RT in Q*r* and *rT*.

From what we have ſaid it will be eaſy to conceive how the Perſpective of any Figure may be put into Perſpective by inſcribing it in a Square, and in the Perſpective of that Square marking the Seat of all the Points in the extreme Parts or Angles

gles of the given Figure; and then, laſtly, by connecting theſe Points with right Lines, the Perſpective of the given Figure is formed.

Thus for Example (in FIG. 3.) let ABCD be the Square upon the Ground Plane, placed directly before the Eye at *(y)*; in this Square are inſcribed the following Figures, (1.) The Square EOFQ with an Angle F in Front; (2.) A Circle circumſcribing that Square, (3.) A Square inſcribed in this Circle in a Situation ſimilar to the original Square; (4.) Another inſcribed Circle; and in that, (5.) an iſoſceles Triangle ZVX. (6.) The ſmall Squares IR, *&c.* in the Angles of the original Square.

Then it is plain, by Inſpection only, how all the Seats of the Points O, I, B, R, E, *&c.* are found in the perſpective Square A*bc*D; and by properly connecting them, how all the Perſpectives of the ſeveral Figures reſult; and even Circles are eaſily traced with an even and dextrous Hand through ſo many given Points. Theſe Figures are readily ſeen and diſtinguiſhed by the correſponding ſmall Letters in each.

The Forms and Shapes of Figures in their perſpective Plan are greatly altered in a *direct View*, but ſtill more ſo in one that is *oblique*, as is evident in (FIG. 4.) where the ſame Square is placed on one Side the Point of Sight *(y)*; and its Perſpective A*bc*D, determined by the Radials, A*y*, D*y*, and the Diagonal DX, contains the Perſpective of the Circle and Square in Front very different in their Forms, Dimenſions, *&c.* from what they had before in the direct View.

If the original Square be divided by equidiſtant Lines perpendicular to the Ground Line AD, as 11, 22, 33, *&c.* their Perſpectives all converge to the Point of Sight *(y)* and the perſpective Line *bc* will be equally divided by them. Any Lines GH, OQ, IK parallel to the Ground Line by transfering their Diſtances to the Ground Line, (that is, making $D1 = DH$, $D2 = DQ$, *&c.*) and then drawing the Diagonals X 1, X 2, *&c.* the Points *h*, *q*, *k*, *c*, will be given in the Radial D*y* through which to draw their Perſpectives *g h*, *o q*, *i k*, *b c*, in this *oblique View*, the ſame as in the *direct one.*

Though Right Lines perpendicular or *oblique* to the Ground Line differ in their Dimenſions in the direct and oblique Views, yet thoſe right Lines that are *parallel* to the Ground Line, are the ſame in both Views. For in the *oblique View* we have $AD : bc :: yD : yc$; and in the *direct View*, it is $AD : bc :: yA : by$; But in the former yD is to yc in the ſame Ratio as yA to yb in the latter; therefore the Ratio of AD to bc is the ſame in both; conſequently, if AD be the ſame in both, then will bc be ſo too.

CHAP.

CHAP. III.

The PRINCIPLES of *Scenographic* PERSPECTIVE, explained and demonſtrated.

HAVING demonſtrated the perſpective Principles of ICHNOGRAPHY, or all that is concerned in the *Ground Plan*; we have, in Effect, done the ſame with reſpect to the SCENOGRAPHY, or all that relates to *Elevations*, or *Upright Structures*, built upon the ſaid Ground Plan.

For whether any Right Line as AD (FIG. 5.) be upon the Ground, or elevated above it, as AB, yet if thoſe Lines are at an equal Diſtance A*y* from the Point of Sight, their perſpective Diminutions will be equal; and they equally reſult from, and may be determined by, the ſame *General Rule* above demonſtrated. For draw the Radials A*y* and D*y*, and the Diagonal AZ, which will croſs the Radial D*y* in *(d)*; and draw *a d* parallel to AD, then will this Line *a d* be the Perſpective of a Line equal to AD, and parallel to it, and at the ſame Diſtance from it. Conſequently A*a d*D will be the Perſpective of a Square upon the Ground Plane whoſe Side is AD.

In like Manner upon the Point D erect the Perpendicular DC = AD, and compleat the Square ABCD; then draw the Radials B*y*, C*y*, and the Diagonal BZ cutting C*y* in *(c)*, and draw *bc* parallel to BC; so shall B*bc*C be the Perspective of the same Square *(viz.* equal to ABCD) placed upon a Ground Plane parallel to AD, and elevated above it to the Height of AB; and at same Distance from it.

By joining the Points *a*, *b*, and *c*, *d*, 'tis evident, the whole Figure becomes the Perspective of a Cube standing on the Ground Line AD, whose Side in Front (or that next the Eye) is ABCD; the Base A*ad*D; the Top B*bc*C; the Right Side A*ab*B; the left Side D*dc*C; and the farthest or hinder Side *abcd*.

What remains is to shew that the Perspective *ab* of a given Line in an Elevation above the Ground Plane is equal to the Perspective of it *ad*, when it coincides with the same. And this is easily seen in the similar Triangles A*y*D, *ayd*, and A*y*B, *ayb*. For from them we have these Ratios A D : *ad* : : A*y* : *ay* : : AB : *ab*; But AB = AD, therefore *ab* = *ad*. And this holds good in very Elevation of the Line AB, since the Angle BAD is not concerned in the above Analogies.

If the Sides of the Cube be continued up to the Line of Sight *y*Z, there will be formed an equal sided *Parallelopiped* upon the same Square Base A*ad*D, but whose upper Surface vanishes into a right Line *e*E*f*F. The Front View of this Solid is AEFD; the Side A*ae*E is called the *Profile* or Side-View, which is greater or less in Proportion to the Obliquity of the View; for *in front*, or a *direct View* it vanishes, as is easy to observe from the Reason of the Thing in FIG. 3, and 4.

If

If the Parallelopiped were carried to a Height above the Line of Sight equal to CF, and was hollow or transparent, then would the under Part of the Top, or highest Surface, have its Perspective exactly equal to B*bc*C; and if it extended equally above and below the Line *y*Z, then the Perspective of the Top will be the same with that of the Bottom, *viz.* A*ad*D.

When it is considered that by having the Distance, Altitude, Angle of Position, and Profile of any Object given, the Perspective Seat of every Point, and therefore of every Line, will be easily formed by the foregoing practical Rule, it will be needless to multiply Words to shew how the Perspective of any Object may be compleatly found by delineating those of the Points and Lines which compose it, as they lie in the Perspective Sides of the Cube or Parallelopiped which circumscribe the Object; As the *Point of Sight* and the *Point of Distance* are always known, and also the *Accidental Points* from the given Position of the Object with respect to the Ground Line, there can remain no Difficulty to the ingenious Artist.

And hence, that which has been hitherto considered as a mighty Difficulty in Perspective, viz. *drawing the Perspective of Objects by Reflection, as those seen in Water*, &c. will be found none at all. For if they are Things in the Heavens, their Altitude measured by a Quadrant gives their Elevation above the horizontal Line HZ; and since, by the Laws of CATOPTRICS the *Angle of the incident Ray is ever equal to that of the reflected one*; therefore the Point of the Image is in a Right Line drawn from the Object perpendicular to the Horizon

rizon HZ, and at the fame Diftance below as the Object itfelf is above it. Hence the Place of the SUN, CLOUDS, TOPS of MOUNTAINS, *&c.* in *Day-Pieces*; and the MOON, STARS, *&c.* in *Night Views*, may be readily and truly drawn in Perfpective, by the fame *General Rule.*

But if the Object OB be upon the Ground, (FIG. 6.) and its Height, and Diftance AB be known, then divide the Diftance AB, in the Point D, fo that it may be BD : AD : : OB : AC = to the Height of the Eye at C; alfo make IB = OB, and draw CI; then will the Angle CDA = IDB = ODB; and confequently, if Water be at D, the Image at I will be feen by the Eye at C. Or a General Rule for finding AD is $\frac{AB \times AC}{OB + AC} = AD$.

But more generally and concifely by *Gunter's Lines* proceed thus; let the Altitude of any Body, on the Earth or in the Heavens, be denoted by the Angle ADC, which fuppofe to be 30 Degrees, and let the Height of the Eye AC be 6 Feet. Then if you extend the Compaffes from 30° to 45° in the *Line of Tangents*, that Extent will reach from 6 Feet to 10,4 Feet in the *Line of Numbers*, which will be equal to AD, the Diftance of the Point D in the Plane of Reflection (or Azimuth of the *Phænomenon*) from the Eye.

The Diftance AD being found in the *Ground Plane*, its *Seat* on the *Perfpective Plane*, or *Picture*, will be found by the Interfection of a *Radial* and *Diagonal*, as above taught.

CHAP.

CHAP. IV.

The ANALYSIS of PERSPECTIVE; or the METHOD of *analysing* a given *Perspective* VIEW or LANDSCAPE.

THE ANALYSIS of PRSPECTIVE consists in the Resolution of a *Perspective Picture* into its *Prototype* or original Component Parts, so that we may form a just Idea of their Situations, Distances, Dimensions, &c. from their Delineation in a given *Landscape*, or Perspective PICTURE.

The first Step to be taken in this Process is to construct two LINES of MEASURES, one for taking the Dimensions, Distances, &c. of Objects upon the *Ground Plane*, and the other for those which have an Elevation above it. The first of these is to be laid upon the Ground-Line (as AG FIG. 5.) and is nothing more than the Division of the lowest Line of the Landscape or Picture into *equal Parts*, which may represent *Feet*, *Yards*, *Rods*, or *Miles*, as the Nature and Extent of the Perspective require.

The second *Line* of *Measures* for Uprights, is to be affixed to the Perpendicular Line GH on the Side of the

the Landſcape. By the Line of Meaſures on the Ground Line AG, the Length of any Perſpective Line parallel thereto may be inſtantly known; as alſo the Height of any Line perpendicular to the Perſpective Plane or Landſcape, in any Part, may be eſtimated by the Line of Meaſures at the Side.

In this Operation we have regard to the Point of Sight *(y)*; and this in any Landſcape is eaſily found by continuing out any two extreme Radial Lines, as AN and GM, till they meet, which will be in the Point *(y)* required. In the Diagram (FIG. 5.) the Scale of Meaſure upon the Ground Line AG contains 15 equal Parts, *Feet*, *Yards*, &c. and the perpendicular Scale GH has eight. And therefore if Lines were drawn by the Edge of a Rule from the Point *(y)* to each of thoſe Diviſions, they would ſhew in any Part of the Ground Plane, or Side Plane, the proper Length of any perſpective Lines parallel to the ſaid Scales, and of Courſe the Form of *Walks*, *Canals*, *Viſtas*, &c. which are perpendicular to the ſaid Scales.

Thus for Example, ſuppoſe a Ruler laid from *(y)* to 2, and by its Edge you draw the Line *n* 2; then will the Space A N *n* 2 be the Perſpective of a Walk or Viſta of 2 Yards in Width, by the Side of the *Perſpective Garden*. And for the ſame Reaſon G M *m* 13 will be ſuch another on the other Side; and 7 *r* R 9 will be the ſame through the Middle of the Area.

But now to reſolve and meaſure any *Parallelogram* in Perſpective parallel to the Ground Line, will require firſt a Determination of the Point of Diſtance Z; for by *Diagonal Lines* drawn from the Diviſions of the

the Scale AG to that Point, the extreme Line AN in the Side of the Landſcape is divided, and by theſe Diviſions the perſpective Diſtances of Objects between any two Parallels to the front Line AG are eſtimated, or become known.

Thus if the Diagonal Z 2 be drawn, it will croſs the Line AN in *t*, and drawing through *t* the Parallel T *t*, it will make the perſpective Walk TA of 2 Yards Width, contiguous to the Ground Line. Again, if Diagonals from Z to 7 and 9 be drawn, they give two Points in the Radial AN at Q, through which two Parallels are drawn, which form the ſame Walk QS, through the Midſt of the Garden. And in like Manner ſuch a Walk is determined at the fartheſt Side MN.

But it is evident, the *nearer* Z is to *y*, the greater will be the Diviſions A *t*, AQ, AN; and leſs, as it is farther removed. Therefore it muſt be determined and fixed for the given Landſcape, which is not always very eaſy to be done. But no Difficulty can ariſe in ſuch perſpective Pieces as have any *regular Figure* in a Side View upon the Ground Line. Thus the two *Summer Houſes* in the perſpective View of CHELSEA-HOSPITAL and GARDENS being of equal Sides, afford a perſpective Square, as A *a d* D (FIG. 5.) the Diagonals A *d* determines the Point of Diſtance Z. Alſo the *Octagon Baſon* in the Court before *Buckingham Houſe*, (now the *Queen's Palace*) will ſhew the ſame Point in the *View* of St. *James*'s *Park*.

To analize a Building or any other Object in any Part of the Landſcape, you are to obſerve, that all the *Ichnography* of it is reſolved and meaſured by the Method juſt mentioned, and the *Scenography* is determined by meaſuring the upright Lines thus: Let the Parallels be drawn for the angular Points, and where they interſect the Side Radial, as ſuppoſe at S, there erect perpendiculars equal to the Heights on thoſe Angles of the Building as S *s*, then the Radial *y* K drawn through *s* will give the Meaſure GK of the true or real Height of the Building in that Part; and thus it will be found for any other.

Hence the Height of *Trees*, *Houſes*, *Men*, &c. are eaſily found on any Part of the Landſcape, and the Manner or Ratio of their Diminution GK, *S s*, ML, as they are removed from the perſpective Plane to the fartheſt Part of the View. Hence alſo it appears that Objects below the Eye riſe conſtantly in the perſpective View towards the *horizontal Line* HZ. And thoſe *above* the Eye in the ſame Manner *deſcend* to it, as they go more remote from the Plane. Thus *Birds* diminiſh to our View, and apparently deſcend towards the Horizon, when they fly from us in a Direction parallel thereto.

CHAP.

CHAP. V.

The MECHANISM of PERSPECTIVE; or *Mechanical* CONSTRUCTION of the THEORY thereof, for a DEMONSTRATION to the SIGHT.

THE MECHANISM of PERSPECTIVE is the next Thing neceſſary for the young Deſigner to be acquainted with, and to have often before his Eyes. It conſiſts in conſtructing or building up the THEORY of PERSPECTIVE into a *real* or *material* FORM, that the *Rationale* of the Science may, as it were, become the Object of Sight, and be thereby much eaſier conceived by the Mind.

In order to this it will be neceſſary to have a rectangular Piece of Wood *(Mahogany* or *Ebony)* about 24 Inches long and 6 or 8 wide; in this at about 10 Inches from the End, is to be made a Dove-tail Groove half an Inch deep and wide, and then a Frame muſt be made to move or ſlide freely in the ſaid Grove, yet ſo as to be always upright or perpendicular to the Surface on which it ſtands. This Baſis or long Parallelogram of Wood will then anſwer to the general Ground-Plane GHIKLM (in FIG. 1.) The Frame to ABCD, the perſpective Plane; and the Groove to the Ground Line OQ.

At or near the End muſt be placed a Stile or Wire repreſented by the Perpendicular HY, with a ſmall Hole in the upper Part at Y, alſo another is to be placed at the other End denoted by LN, with a Perforation at N.

In the Frame AC muſt be fixed a large Piece of tranſparent Horn, ſuch as is uſed for *Lanthorns*. Then having a Square made of a thin Piece of Braſs repreſented by QOTR, with fine Holes in the Angles and Sides at Q, P, O, R, S, T, and the other Parts when neceſſary; the Perſpective of this Square is to be drawn upon the *Plate of Horn* with a ſharp Steel Point, in the Manner ſhewn in FIG. 2.

This being done, through the ſeveral Points in the Perſpective Square as at Q, P, O, *T*, *s*, *r*, *g*, *h*, and *y*, let ſmall Holes be drilled, and then the Horn being fixed in the Frame AC, that muſt be ſo placed in the Groove Q O that the *Ground Lines* of both the *Original and Perſpective Plane* may coincide as near as poſſible.

Then for the *Viſual Rays*, they will be very properly repreſented by *fine Threads* fixed in and paſſing through the Holes in the Stiles and Planes, *&c.* Thus if upon the Board GK, be drawn the parallel Lines GM, IK, upon Paper, Paſteboard, Ivory, *&c.* as alſo the Lines GI and MK: And alſo if Threads fixed in G and M be put through the Holes in the Stiles at Y and N, and then carried down and fixed in the Points I and K;

K; and, laſtly, if another Thread paſs from Y through the Point *(y)* in the Horn, and be fixed in the Hole at N; then will the *perſpective* PRISM, be properly conſtructed.

If the Line OR be continued out upon the Board to *(a)* and a Thread be fixed in that Point, and from thence carried through a Hole in the Point of Diſtance X in the Horn to the Point Y in the Stile, and there fixed, you will have the Plane O*a*XY conſtructed for the Demonſtration of what relates to the Perſpective of *Diagonal Lines* and their *Parallels*.

Laſtly, let Threads be fixed in the Holes of the Square OQRT, and carried through the Holes in the Plate of Horn, and all of them fixed tight in the Hole Y of the Stile HY, then will the Square *Pencil of Viſual Rays* be properly formed for ſhewing the Reaſon of every Thing which concerns the Perſpective of *Radial Lines*, or thoſe which are *perpendicular* and *parallel* to the *Ground Line*; eſpecially if to theſe are added other Threads from Holes in the Line OT continued out, and paſſing through correſponding Holes in the Horn which will determine the Perſpectives or Landſcapes of any given *Parallelograms*.

All that has been hitherto ſaid relates to the *Mechanical Conſtruction* of the THEORY; but the *Practical* Part of PERSPECTIVE may alſo be proved and confirmed by Experiments made in the ſame Machine with a little Variation; for in this Caſe there will be no Need of Threads at all; and inſtead of a Stile HY in Form of a Wire, it ſhould have a round Plate

Plate of Braſs on the Top, with a ſmall Hole in the Middle to look through, like a common *Sight-Vane* to Quadrants, *&c.* Then if ſeveral Pieces of Horn (very ſmooth and tranſparent) were fitted to the Frame AC, ſo as to be taken in and out at pleaſure, and upon theſe ſeveral Perſpectives of Object drawn, as of a *Chair*, *Table*, *Inſide* of a *Room*, *Doors open*, &c. Then if theſe are ſucceſſively, and appoſitely placed by the Ground Plane of the Original, and viewed through the Sight Vane at Y, the *Prototype* and its *Perſpective* will be ſeen perfectly to coincide upon the Plate of Horn in every Point and Part; and conſequently the *Truth of the Draught* is eſtabliſhed.

In this laſt Experiment inſtead of *Horn*, a Plate of *poliſhed* GLASS would do much better, as no Holes thro' it are neceſſary, were it not for two Objections; firſt the Lines or Landſcapes are not ſo eaſy to draw upon Glaſs, unleſs by an accuſtomed and ſkilful Hand; and ſecondly, when well drawn, are liable to be broke and deſtroyed. And, indeed, ſome Plates of Horn may be found but little inferior to Glaſs, and will therefore render it leſs neceſſary.

However, it will be proper here to obſerve, That the Perſpective of Objects by *Reflection* is the ſame with that by *Refraction*, as will appear from the Reaſon of the Thing; for, let SPH be the Ground Line (FIG. 7.) P*y* the Section of the Perſpective Plane; S the Place of an Object, from which a Ray S *s* falling upon the Plane in *(s)* is reflected to the Eye at *(y)*, and continuing *y s* to H, the Point H will be the apparent Place of that Object, and PS will be equal to PH, by the common

mon Principle of Optics *that the Angle of Incidence* S*st* *is equal to the Angle of Reflection* *tsy*; But S*st* = *s*SP, and *tsy* = *s*HP; therefore the Effect, or Senfation in the Eye, is the fame whether it be feen by *Reflection from* S, or *by Refraction from* H, and the Perfpective Seat (*s*) is the fame upon the Plane in both Cafes.

Therefore if (in FIG. 2.) ABCD reprefent a plain LOOKING-GLASS (fuch as is ufually placed over the Chimney-Piece in a Parlour) then all Objects expofed to it are viewed in *true Perfpective* upon it. And if the Line of Sight EF, Height of the Eye P*y*, and Points of Diftance X, Z, were drawn upon it with Ink, or black Varnifh, and OQRS a Square laid horizontally before it, then drawing the Radials O*y*, Q*y*, and the Diagonals OX, QZ, your will have the Perfpective of that Square, *viz.* QO*Tr* upon the Face of the Glafs the very fame as by Refraction in FIG. I.

It will be very eafy to underftand that the Theory of Perfpective may be conftructed for a perfpective Plane that has any given Inclination to the Ground Plane; fince the Diftance of the Eye from the Plane is, in this Cafe, *equal to the perpendicular Diftance from the Ground Line and Co-tangent of the Plane's Inclination added together, the Radius being the Height of the Eye*; and all other Parts of the Procefs will be the fame, or fimilar to thofe above defcribed.

CHAP.

CHAP. VI.

The CONSRUCTION and USE of a *Perspective* TABLE, for readily drawing in true PERSPECTIVE any natural SCENE, or VIEW of OBJECTS proposed.

THE Method of putting any Objects into a Perspective View by practical Rules derived from the Theory, as above delivered and explained, requires the Knowledge of their various Dimensions, and Distances, and is a Work of Time, with many other Circumstances, the Consideration of all which together has made it necessary to contrive still more practical and expedient Methods of delineating any given Object, or Groupe of Objects, in true Perspective.

And this has been found as easy and practicable by the Invention of a *Perspective* PLAIN TABLE, as can be desired; and of these Tables, there have been many Forms and Contrivances since that first invented by Sir. C. WREN. They differ only in Form and Construction, the Principle of the Operation is the same in them all. That which I have here

here given in a Diagram admits, I think of the greateſt Facility and Certainty of Motion, which is the utmoſt Excellency of this Sort of Inſtruments.

This *Perſpective drawing* TABLE is repreſented in FIG. 8. It may conſiſt of a Board, with a Rectangle or Frame ABCD, to take off and put on, upon Occaſion, whoſe Uſe is to hold firmly a ſmooth Sheet of Paper EFGH upon the Body or Plane of the Table in the ſame Manner as in the *Plain Table* for SURVEYING. But this *perſpective Table* may be made entire, without ſuch a Frame, with proper Springs on the Sides to hold the Paper for the Draught.

This Table placed upon a Foot and Pillar ST, may be put into a *vertical Poſition*; it is not neceſſary it ſhould be perfectly perpendicular, and therefore requires no Plumb-Line or Spirit-Level.

The Table being fixed pretty ſteady, you next apply, or fix to it, a Set of three moveable Quadrangles ABIK, IKLM, LMNO, which have all a perfect Freedom of Motion at their reſpective Angles I, K, L, M, N, O; Theſe are connected with the lower Part of the Table by the Ends A, B, going into proper Grooves or Holes with Spring-Catches, for holding them very faſt. In order to make them as light as poſſible, it will be ſufficient to have the whole Mahogany, except the four Sides IM, KL, LN, MO, which may be ſmall Slips of Braſs.

When Things are thus diſpoſed, as in the Diagram they appear, it is plain the Part NO of the upper Quadrangle, is freely moveable over all the Plain of the Table, and therefore in the Middle at P is ſcrewed faſt on, a Stile or perpendicular Piece PQ, with a Point at Q called the *Index* of the Stile or *Tracer*.

This Tracer PQ has in its lower Part at P a Pencil fixed in the ſcrew, by which it is faſtened to the Piece or Bar NO. Now let *(abcd)* be an imaginary tranſparent Plane, as in the Theory (FIG. 1. and 2.) upon which let us ſuppoſe the *Perſpective* of a *Houſe*, *Trees*, *Walls*, &c. to be delineated; and that this Plane is placed upon the upper Part of the Table CD.

Then it is evident, if the Hand holding the Pencil at P, were to move in ſuch Manner as that the Index Q might be made to paſs along or deſcribe any Line on the perſpective Part, the Pencil itſelf muſt neceſſarily deſcribe a Line of the ſame Length, and in the ſame Poſition, upon the Paper on the Table; and conſequently, by this eaſy Artifice, all the *Contours* or Out-Lines of the *Landſcape* will be readily delineated upon the Paper of the *Perſpective Table*.

Again it is evident, that the Eye placed at ſuch a Height and Diſtance from the Plane *(abcd)* as belong to the Landſcape drawn upon it, will ſee all the Lineaments of Objects in the ſaid Picture coincide exactly with thoſe of their *Prototypes* or Originals, by the viſual Rays paſſing to the Eye, from correſponding Points in each; Therefore it will follow, that if the ſaid perſpective Table *(abcd)* were removed, and a *Sight-Vane* fixed in the proper *Point of Sight*, an Eye looking through it, would perceive the Index

Q

Q paſs over the ſame Lines in the original Objects, as before in the Landſcape, if moved in the ſame Manner; and conſequently the Pencil at P will deſcribe the ſame Lines as before; and thus a true *perſpective Draught* upon the Table ABCD will be made of any diſtant Objects propoſed.

This Practice is univerſal for all Objects remote or near, for the *Sight Vane* being moveable, may be fixed at any Diſtance from the Table, and Height above the Horizon, and conſequently may be thereby adapted to any Diſtance of Objects, or Size of the *Drawing* propoſed; for from a bare View of FIG. 2. it is evident *that the perſpective Plane* QO *t r or Landſcape will be larger as the Height of the Eye* P*y is greater, at the ſame Diſtance y* X.

Alſo it is as evident, *that for the ſame Height of the Eye y* P, *the Landſcape will increaſe or decreaſe as the Diſtance of the Eye from the perſpective Plane decreaſes or increaſes.*

Hence when the Eye is at a great Height, and the Diſtance of the Plane or Table from it very ſmall, the Perſpective of a given Square will be *enormouſly great*, and almoſt of a *triangular Form*. Therefore a *Face*, *Head*, *&c.* drawn in the Square will be greatly *deformed* when delineated on the perſpective Draught of the Table. But this very deformed Image will appear to an Eye at that given Height and Diſtance as formous and perfect as the Original. And thus the whole Doctrine of the *Anamorphoſis* or *Deformation of Images* is elucidated, as it were, by Inſpection.

The DOCTRINE of SHADOWS is reckoned a PART of PERSPECTIVE; but it is rather only the *reverſe* of the direct Method; for if (in FIG. 1.) we ſuppoſe a CANDLE placed at Y, at the Diſtance HP from the perſpective Plane AC, and if OQ *rt* repreſent an *Opake Object* upon that Plane, then it is evident that Rays of Light proceeding from the *Candle*, will project the SHADOW of that Object into a perfect *ſquare Form* OQRT on the *Ground Plane* GK. So that *no new* PRINCIPLES are required for a further Proſecution of this Subject; and a ſuperfluous *Copia Verborum* is not neceſſary to the ingenious young Student in this Art.

CHAP. VII.

The CONSTRUCTION and USE of a *Double Parallel* RULER in PERSPECTIVE, ARCHITECTURE, &c.

AS no Inſtrument is of ſo great and general Uſe in all Kinds of Drawing in Architecture, Perſpective, &c. as the PARALLEL-RULE, I ſhall here give the Conſtruction and Deſcription of one which is quite New, and better accommodated to perſpective Drawing and Deſigns in parti-

particular, than thoſe of the common make. This may be properly called the *Double Parallel Ruler.*

The Conſtruction of this Ruler is immediately derived from the Mechaniſm and Contexture of the two Quadrangles IKLM, and LMNO; and is in reality nothing elſe, excepting that the Parts or Bars IK and NO are of double the Length, continued out at the Ends K, and N; and the Middle or ſmall Bar LM made longer at the End M. All this is evident in the Form of the Ruler itſelf (FIG. 9.) a little Way opened, where the ſame Letters denote the ſame Parts in both Figures 8 and 9.

The Deſign of this Parallel-Rule is to command the whole Extent of the perſpective Table without being moved; that is, one Side of the ſaid Rule IK (FIG. 9.) being fixed by two *ſteady Pins* to the lower Part of the *Frame* EF, the other Bar or Side of the Ruler ON is capable of being opened parallel-wiſe over the whole Table, or quite to the Top GH; or from the Side FG to the Side EH, at one Opening.

This Opening of the Ruler is not only *parallel*, but *perpendicular* likewiſe; or the Ends of the moveable Bar are always perpendicular over the Ends of the fixed one. This evidently follows from its being a *double Ruler*, or conſiſting of two ſingle ones connected, *viz.* IKLM and LMNO (FIG. 8.) For as much as the End L over-runs the End K in the firſt, juſt ſo much muſt the End O go out beyond M in the ſecond; conſequently the Points O and N will be vertical to I and K in the whole Operation.

From

From this Conftruction of the Double Parallel and Perpendicular Ruler, it appears that the Extent of the Opening will be equal to twice the Length of the Side-Pieces IM or MO (FIG. 8.) that is, nearly the Length of the Ruler itfelf as reprefented in FIG. 9.

Hence it is plain, that by this Contrivance all Lines in the perfpective Drawing on the Table which are parallel to the Sides, may be examined, continued, and compleated without the leaft Trouble, and with the greateft Certainty and Truth. Alfo all other Lines in the Drawing are commanded at one opening and placing the Ruler; and fo all the conftituent Lines being drawn, the whole Perfpective or Landfcape is finifhed and compleated in much lefs Time, and with greater Eafe and Truth, than it could be by the *Common Parallel-Rules.*

The PRINCIPLE, therefore, and *Rationale* of the PERSPECTIVE TABLE and Double PARALLEL-RULER are the fame in each; and it would be more prolix than difficult to fhew how very ufeful fuch a *plain* DRAWING TABLE and PARALLEL RULER would be in all Branches of practical Mathematics, and particularly in drawing the various ORDERS of ARCHITECTURE, EDIFICES, and DESIGNS of all Kinds.

I fhall conclude with one Obfervation more, and that is, that fince the End O of the moveable Bar NO always flides along the Side EH of the Table, it is evident that if a *Line of* Inches (or any other *equal Parts)* were placed upon the Side EH, and a proper *Nonius* upon the End of the Ruler at O, that then *parallel Lines* might be drawn at any given Diftance from each other,

other, to the 100th or 400th Part of an Inch, if required. Hence there will be no need to expatiate on the extreme Utility of the Ruler thus improved in drawing the *parallel Lines at their requisite Distances*, which constitute the several Members and Ornaments of PILLARS, PILASTERS, and their PEDESTALS, &c. in every Order, and according to the Measures proposed by the most celebrated *ancient or modern* ARCHITECTS.

CHAP. VIII.

A Demonstration that PERSPECTIVE is only a Branch of the SCIENCE of OPTICS; and that a PERSPECTIVE PICTURE is the same THING as a *Landscape* formed in the Focus of Optical Glasses, whether MIRRORS or LENSES.

THAT PERSPECTIVE is only one particular Case of OPTICS is evident to every one who understands the Principles of that Science. But for the Satisfaction of such as are not Proficients in the Mathematical Part of Optics, I shall here subjoin an easy and concise Demonstration of the above

above Poſition, deduced from the well known Principles of OPTICS.

In order to this, let IG (FIG. 10) be a *convex* SPECULUM, expoſed to a given Object AB, at a given Diſtance IA, and let N be the Center, O the geometrical Focus, and IN the Radius of the ſaid Mirror; laſtly, let *(ab)* be the Image of the Object AB formed by the Mirror; *then their Proportion will always be that of their Diſtances from the Glaſs*, viz. AB : *ab* : : AI : I*a*.

Again it is demonſtrated by Writers on Optics, that $IA \times IN = Ia \times \overline{2IA + IN}$; Hence by this Equation and the foregoing Analogy, we get $AB \times IN = ab \times \overline{2AI + IN}$, which gives this Analogy, *viz.* AB : *ab* : : 2 IA + IN : IN.

In like Manner if CD be any other Object, at the Diſtance IC from the Mirror, and *(cd)* be its Image, then it will be $CD \times IN = cd \times \overline{2ID + IN}$; If the Objects AB and CD are equal, then we have $ab \times \overline{2AI + NI} = cd \times \overline{2CI + IN}$; and from thence, this Analogy, *ab* : *cd* : : 2 CI + IN : 2 AI + IN.

Now when Objects are at a great Diſtance, the Radius of the Speculum will be inconſiderable, and may therefore be neglected in the laſt mentioned Analogy, which will then become *ab* : *cd* : : 2 IC : 2 IA : : IC : IA; that is, *The Images of all equal Objects at different, but great Diſtances from the Mirror, are inverſely as thoſe Diſtances, in their linear Dimenſions.**

And

* Or more conciſely thus; AB : *ab* : : AI : I*a*; and CD : *cd* : : IC : I*c*; therefore when AB = CD, we have $\frac{ab \times AI}{Ia} = \frac{cd \times IC}{Ic}$; but at great Diſtances, it is $Ia = Ic$, nearly; then $ab \times AI = cd \times IC$; whence *ab* : *cd* : : IC : IA, as above.

And this is the very Proportion in which the *Perſpectives* of all Objects encreaſe or decreaſe upon the perſpective Table or Plane, as is thus eaſily ſhewn. Inſtead of the Glaſs, let the *Point of Sight* or Place of the Eye be at I, viewing the Objects AB, CD, through the perſpective Plane EF; and by drawing the Viſual Rays IB, ID, they will define the Perſpectives of theſe Objects upon the Plane, *viz.* E*e*, E*h*; Then becauſe in the ſimilar Triangles IBA, and I*e*E we have AB : E*e* :: IA : IE; and the ſimilar Triangles ICD and IE*h*, give CD : E*h* :: IC : IE; conſequently if AB = CD, we have E*e* : E*h* :: IC : IA; that is, *the perſpective Appearances of equal Objects are inverſely as their Diſtances from the perſpective Table*; and therefore, *they are perfectly ſimilar to thoſe Images which are formed of diſtant Objects by optical Glaſſes.*

What has been now demonſtrated for a *convex Speculum*, is ſhewn in the ſame Manner for a *concave One*; as alſo for a *convex Lens* of any Kind; wherefore it is very evident, *that the whole Science of Perſpective is only one particular Caſe of Optics*, viz. *that where the Picture of Objects, at a great Diſtance, is repreſented to the Eye by the Glaſs.*

From this View of Perſpective, ſeveral curious and intereſting Problems will admit of a Solution in this Science, which do not occur in Treatiſes on this Subject; for it will be eaſy from what has been premiſed, to determine in what Caſe the Image or Landſcape on the perſpective Table, is equal to that in the convex Mirror or Lens. Becauſe, in this Caſe, we have $\frac{AB \times IE}{IA}$ = E*e* in Perſpective; and in the convex Mirfor it is $\frac{AB \times IN}{2AI + IN} = ab$; therefore ſince, by Suppoſition, $Ee = ab$, we have $\frac{IE}{IA} = \frac{IN}{2AI + IN}$; conſequently $IA + IN = IE \times \overline{2AI + IN}$.

From this Equation three different Problems will ariſe, *viz.* (1.) Given the Radius of the Mirror IN and Diſtance of the Object IA, to find the *Diſtance of the perſpective Plane* IE, which will be $IE = \frac{IA \times IN}{2IA + IN}$. (2.) Given the Diſtance of the Object, and of the perſpective Plane, to find IN the *Radius* of the Mirror, we have $IN = \frac{2IA \times IE}{IA - IE}$. (3.) Given the Radius of the Glaſs, and Diſtance of the perſpective Plane, to find the *Diſtance of the Object*, which will be $IA = \frac{IE \times IN}{IN - 2IE}$.

Hence alſo it appears, that when $IN = 2IE$, then IA will be infinite, or that when the Diſtance of the perſpective Table is equal to half the Radius of the convex Mirror, *then the Pictures or perſpective Appearances of Diſtant Objects muſt be the ſame in each*, as ſhewed before.

After the ſame Manner Theorems are deduced for the Solution of the ſame Problems with a convex Lens, whoſe Radius, or focal Diſtance, is IN; for in this Caſe we have $IA \times IN = IE \times \overline{IA - IN}$; and therefore any two of the three Quantities IA, IE, IN, being given, the other is known from the Equation.

From what has been demonſtrated, the great and excellent Uſe of *convex* MIRRORS and LENSES to all *Drawers*, *Deſigners*, *Painters*, &c. muſt evidently appear; as they exhibit Pictures of Objects in all their native and poſſible Perfection, and indeed far beyond the

Power

Power of Art or Genius of Man to equal; but ſtill this is no Reaſon why they ſhould not be copied and imitated as *Patterns* and *Originals*; As all artificial Drawings, though by the moſt exquiſite Pencils, are nothing but Reſemblances of ſome particular Scenes of Nature ſuppoſed to be formed by theſe Glaſſes. But there is ſomething peculiar in the Nature and Effect of each, which may be uſeful to the young Draughts-Man to know.

The *convex* SPECULUM preſents to the Eye a View of the moſt perfect and Genuine LANDSCAPE of all diſtant Objects to which it is expoſed; this Picture is always *erect*, and therefore in a fit Poſition to be copied; all Objects are here projected according to the moſt perfect Laws of Perſpective; the *Contours* or Out-lines, *Light* and *Shadow*, and all other Pictureſque Incidents, are *here* what the young Deſigner ſhould endeavour to impreſs his Mind and Memory with, in the ſtrongeſt Manner, as being the Standard of all Perfection in his Art; and the only Pattern for forming juſt Ideas of every Part thereof. For it will be a Maxim that will ever hold true, *that Painters will always excel in Proportion as they imitate Nature more truly.*

The *convex* LENS is likewiſe of great Uſe to Deſigners, as it preſents them with a *real Picture* or *Landſcape* of Objects, and not one that is merely *ideal* or *pictureſque*, as in the foregoing Caſe. This Landſcape by the *Lens* is actually formed upon the Paper before your Eye, and may be readily copied or traced out by the Hand of the Artiſt; and for this Purpoſe the portable *Camera Obſcura* is the moſt valuable Inſtrument, as it immediately furniſhes an original Picture of any propoſed Scene or Group of natural Objects ready for the Hand to imitate and copy into a Landſcape. From all which it appears that both Art and Nature have abundantly contributed to render the *Art of* PAINTING as perfect as poſſible; if the Profeſſors of that Art be negligent in the Uſe of ſuch Means, that is another Caſe, though but too common.

From theſe Premiſes it likewiſe follows, that the young Artiſt may have it in his Power *to draw any Object in any Proportion leſs than the Life*; for ſuppoſe the given Proportion of the Object to its Image be that of m to 1, then from the above Theorem for the *convex* MIRROR we have $\frac{m-1}{2} \times IN = IA$; and in the *convex Lens*, we have $\overline{m+1} \times IN = IA$; Therefore by knowing the Radius IN of the Mirror or Lens, you have the Diſtance IA, at which the Glaſs muſt be removed from the Object, ſo that its Image may be leſs in the Proportion of 1 to m.

For Example, ſuppoſe it be required to draw an Object four times leſs than $m = 4$; and ſuppoſe the Radius of the Mirror IN $= 24$ Inches, $\frac{m-1}{2} \times 24 = \frac{3}{2} \times 24 = 36$ Inches, the Diſtance IA at which the Object muſt be placed from ſuch a Mirror that its Image may be juſt a *fourth Part* as long and as broad.

But in Caſe of a Lens whoſe Radius (or focal Diſtance) is IN $= 24$, as before, you have $\overline{m+1} \times IN = 5 \times 24 = 120$ Inches, the Diſtance of the Object from the Lens, that it may be four times leſs.

If the Image is required equal to the Object, then $m = 1$; therefore IA $= 0$, or the Object muſt touch the convex Mirror, becauſe in that Caſe $\frac{m-1}{2} = 0$. But

in

in the Lens we have $\overline{m + 1} = 2$, which ſhews that 2 IN = IA, or that the Object muſt be placed at twice the focal Diſtance of the Lens to have an Image equal to itſelf. And hence the excellent Uſe of the PROPORTIONAL CAMERA OBSCURA, *contrived for readily producing an Image in any given Proportion leſs than the Object.*

We have not yet mentioned the *concave* MIRROR, tho' it muſt be allowed to have the greateſt Variety of curious Effects of any Optic Glaſs whatever. Among others, *this will preſent to the Eye an Image of an Object, not only in any Degree leſs, but alſo larger than the Life.* The Theorem here being $\frac{m + 1}{2} \times IN = IA$, where 'tis evident, that when the Image is required to be made equal to the Object, or $m = 1$, then IA = to IN, or the Diſtance of the Object and its Image both coincide with the *Radius of Concavity.*

The principal Uſe of a *concave* MIRROR, *is in reverſing a Landſcape, or any perſpective View,* that is in giving a *Relievo* to the Picture, or ſetting every Thing upright, or in its natural Poſition. This it muſt neceſſarily do, becauſe the Effect of a *concave Mirror* is juſt contrary to that of a *convex one*; and therefore as the *convex Mirror* projects all upright Objects *in Plano*, in a Pictureſque Landſcape, ſo the *concave Mirror* will reverſe that Picture, and raiſe the ſeveral Objects from the Plane to their natural upright Poſition.

Hence a perſpective View of the CITY of LONDON (for Inſtance,) being delineated from a Picture thereof in a convex Mirror, would when placed before a concave Mirror of the ſame Radius, preſent to the Eye a delight-

delightful Scene of the City, and all its CHURCHES, TOWERS, BUILDINGS, &c. erect, the fame as it appears to the naked Eye at the *fame Diftance and Height above it* as the convex Glafs was placed in.

It is no Wonder that the Effects of optical and perfpective Glafs are fo naturally pleafing and delightful to every ingenious Mind, when it is confidered, that the very Principles upon which they depend are founded in abfolute HARMONY itfelf; for the original Equation above mentioned, *viz.* $IA \times IN = Ia \times \overline{2\,IA + IN}$ gives this Analogy, $AN : IA :: aN : Ia$; and therefore the Line AN *is always divided into mufical or harmonical Proportion, in the Points* A, I, *a*, N, that is, *by the Place of the Object, the Place of the Mirror, the Place of the Image, and the Center of the Speculum, whether convex or concave,* I thought it was neceffary the young Artift fhould be acquainted with fo noble a Principle in the Theory of his Science, efpecially as all Authors of Perfpective have been filent on this Head.

CHAP. IX.

The PRINCIPLES of *Spherical* PERSPECTIVE demonſtrated, and applied univerſally to the *Geographical* and *Aſtronomical* PROJECTIONS of the SPHERE in *Plano*, for the *Conſtruction* of MAPS, PLANISPHERES, ANALEMMA, &c.

THE PRINCIPLES of *ſpherical* PERSPECTIVE are next to be explained; and from thence it will appear, that they are the true Baſis of all the Variety of PROJECTIONS of the SPHERE *in Plano*, that are ſo common in aſtronomical and geographical Treatiſes; and not only ſo, but it will be manifeſt that the Doctrine of *Spherical Projections*, which treated in the common Way is very intricate and confined, will, by the RULES *of* PERSPECTIVE, be rendered facile and univerſal.

To this End it will be neceſſary to conſider the Conſtruction of a Figure conſiſting of *three Planes* (as FIG. 11.) of which one is the *perſpective Plane*, or *Plane* of *Projection* AB. The ſecond is CD in a vertical Poſition to the Horizon, and is therefore called the *vertical Plane*. The third is the Plane EF, which paſſes through the Eye at I, and being parallel to the Horizon

zon is called the *horizontal Plane.* These Planes are all supposed to intersect each other at right Angles; Whence the Intersection MN is called the *horizontal Line*; and OP the *vertical Line.* And, lastly, the Line IL is called the *Principal Ray*, as coinciding with the Axis of the Eye.

Let H be a given Point, whose perspective or Seat (*h*) is to be determined upon the perspective Plane AB; from the Point H let fall the Perpendiculars HK to the vertical Plane, and HG to the horizontal Plane; and draw the Lines IK, IH, IG, which will pass thro' the Plane of Projection in the Points *k*, *h*, *g*. Then the Position of the Point *h*, in the Plane of Projection AB, with respect to the vertical and horizontal Lines OP, and MN, is thus determined.

The Planes being all given in Position, their intersecting Lines MN, OP, are given in Position on the Plane of Projection. Also the Position of the Line IL is given; and because the Position of the Object H is given, its Distances HK from the vertical Plane, and HG from the horizonal Plane are known; also its Distance *l*L from the perspective Plane, as likewise that of the Eye I*l*, are both known. Then in the similar Triangles ILK and I*lk* we have IL : I*l* : : IK : I*k*. And the similar Triangles IKH, I*kh*, give IK : I*k* : : HK : *hk*; consequently we have from both Analogies, IL (= I*l* + *l*L) : I*l* : : HK : *hk*; which in Words is

GENERAL RULE I.

As the Sum of the Distances of the Eye and the Object from the Plane of Projection is to the Distance of the Eye

Eye from that Plane, So is the Distance of the Object from the vertical Plane, to the Distance of its Seat or Picture from the vertical Line.

And in like Manner, we have IL : Il : : HG : hg; which gives in Words

GENERAL RULE II.

As the Sum of the Distances of the Eye and Object from the Plane of Projection, is to the Distance between the Eye and this Plane, so is the Distance of the Object from the horizontal Plane, to the Distance of its Picture from the horizontal Line.

It is manifest that if the Object be placed between the Eye and the perspective Plane, these Proportions will be the same, only using the Word *Difference* instead of *Sum* in the first Term of each.

To apply what has been premised, it is evident that in Projections of the Sphere, the above mentioned three Planes may be applied, but they will be of a *circular Form*; and then by these Rules, any Point upon the Surface of the Globe or Sphere will have its Seat ascertained in the Plane of Projection, for any Position or Distance of the Eye whatever. But all Distances in this Case must be estimated in Parts of the *Radius* to a *Table of Sines*, as will be easy to understand by the following Examples.

Let it be required to find the Perspective or Seat of any Point upon the Surface of the Globe projected upon the PLANE of the EQUATOR, as seen by the Eye at a given Distance in the Axis of the Globe produced.

G The

The three Planes are in this Example, (1.) AB the *Plane of Projection*, or that of the *Equator*. (2.) CD the *vertical Plane*, which is that of the *Equinoctial Colure*. (3.) EF the *horizontal Plane*, which is that of the *solstitial Colure*. For these three Planes are all at right Angles to each other.

The Place of the Star upon the Globe being given by a Table of Right Ascensions and Declination, we shall have (by the *Doctrine of the Sphere*,) this Analogy, *viz.*

As the Radius is to the Sine of Right Ascension, so is the Co-sine of Declination *to the Star's Distance from the vertical Plane.*

And then again,

As Radius to the Co-sine of Right Ascension, so is the Co-sine of Declination to the Star's Distance from the horizontal Plane.

The Distance of the Star from the Plane of Projection, is the *Sine of its Declination*, and the Distance of the Eye from the said Plane is to be expressed by some Multiple of Radius, as nR; the Value of (n) being taken at Pleasure, and R = 10000 = *Radius*.

Thus let the Place of ALDEBARAN or the *Bull's Eye* be required in a PLANISPHERE upon the PLANE of the EQUATOR; The *Declination* of this Star is 16 Degrees North; the Sine of which is 2756, therefore that is the Distance of *Aldebaran* from the *Plane of Projection*.

The

The Right Aſcenſion of this Star is 65° 30′, therefore ſay, As Radidus — — 10,
To the Sine of Right Aſcenſion 65° 30′ 9,959023
So is Coſine of Declination, 74 0 9,982842

To the Star's Diſtance from the *vertical Plane* —— } 8747—9,941865
Then again, As Radius —— —— 10,
To the Coſine of Right Aſcenſion 24° 30′ 9,617727
So is Coſine of Declination, 74 0 9,982842

To the Diſtance of the Star from the *horizontal Plane* — } 3986,2 — 9,600569

Having the Diſtance of the Star from the three Planes, and the Diſtance of the Eye nR = 10000 n. You ſay (by General Rule I.) As 10000n + 2756 : 10000 n : : 8747 : $\frac{n \times 87470000}{10000n + 2756}$ = Diſtance of *Aldebaran* from the vertical Line in the Planiſphere.

Then (by Gen. Rule II) ſay

As 10000 n + 2756 : 10000 n : : 3986,2 : $\frac{n \times 39862000}{10000n + 2756}$ = the Diſtance of the Star from the horizontal Line in the Projection.

Thus you have the Place or Seat of the Star in the Planiſphere for any Diſtance of the Eye whatſoever. If we put $n = 1$, then is the Eye upon the Surface of the Globe, *viz.* in the *South Pole* of the World, and the *Diſtance of Aldebaran* from the *vertical Line* will be $\frac{87470000}{12756}$ = 6857,3 of ſuch Parts as the Radius of the Planiſphere contains 10000.

 Again,

Again, the Diſtance of the Star from the *horizontal Line* is, in this Caſe, $\frac{39862000}{12756} = 3125$. And thus the Places of all the Stars are put down in a Celeſtial Planiſphere according to the common *Stereographic Projection* upon the *Plane of the Equator*.

The Stars may in the ſame Manner be projected upon the PLANE *of the* ECLIPTIC, and thus from the PRINCIPLES of PERSPECTIVE, you ſee the Reaſon and Foundation of thoſe two excellent celeſtial Projections of the Conſtellations upon the Planes of the ECLIPTIC and EQUATOR publiſhed ſome Years ago by Mr. SENEX.

If the Surface of the Terreſtrial Globe were to be thus projected upon the Plane of the Equator, the Places muſt be put down by the ſame Rules, but inſtead of *Declination* you take the *Latitude* of the Place; and for *Right Aſcenſion* you take the *Longitude* from the *firſt Meridian*, or *vertical Plane*. And by this Means you will have the old *Ptolomaic* MAP *of the* WORLD, in the Center of which is the *North Pole*; the *Meridians* are all *right Lines*; and the *Parallels of Latitude* are all *concentric Circles*.

The common MAPS of the WORLD, are Projections upon the Plane of the *firſt Meridian* taken at Pleaſure; the vertical Circle is the *Plane of the Meridian* at 90 Degrees from it; and the Horizontal Circle is the *Plane* of the *Equator*; and in the Interſection of the two laſt, the Eye is placed at the Diſtance nR, as before.

Thus for Example, let the MERIDIAN of LONDON be the Plane of Projection, and let it be required to find the

the Seat of the City of CONSTANTINOPLE in the MAP; the Latitude of this City is 41°, and Longitude East 29°. Then the Distance from the *horizontal Plane* is the *Sine* of the *Latitude*; and since the Difference of Longitude between this City and the vertical Plane is 61°, you say,

As Radius to the Sine of the Difference of Longitude, so is the Cosine of the Latitude to the Distance of Constantinople *from the vertical Plane.*

And again,

As Radius to the Cosine of the Difference of Longitude, so is the Cosine of the Latitude to the Distance of the City from the Plane of Projection.

Having thus found the Distances from the three Planes, you have the Distance of *Constantinople* from the *vertical Line* by the first General Rule of Perspective; and the Distance from the *horizontal Line* by the second; and therefore you have its true Place in the *Plane of Projection.*

If we put $n = 1$ as before, then the Eye is upon the Surface of the Globe, *viz.* in the Pole of the opposite Hemisphere; and the MAP becomes that of the vulgar *Stereographic Projection.*

If the Place be in the Equator, its Distance from the horizontal Plane is $= o$; and its Distance from the vertical Plane is the Sine of the Difference of Longitude, which put $= s$; and the Cosine thereof $= C$; then by Rule I, we have $nR + c : nR :: s : D =$ *Distance from the vertical Line*, in the horizontal Line of the Map, and when $n = 1$, it will be $\frac{Rs}{R+c} = D$. Let $s =$ Sine

s = Sine of 10 Degrees, then D = 8749. But if s = Sine of 80°, then D = 8391,4 which taken from Radius = 10000, leaves 1608,6; therefore the Width of 10° of the Equator in the Middle of the Map will be to the Width of 10° at the Side, as 875 to 1608 $\frac{1}{2}$. Consequently it appears how much *such Maps* must differ from the Truth of GLOBES.

Let the Radius, or half the horizontal Line, be divided into a Number of equal Parts, denoted by (p,) and let m denote any Number of them; then will $\frac{mR}{p} = \frac{nRs}{nR + c}$, whence we have $n = \frac{mc}{sp - mR}$. Hence if p = 90, and m = 10, 20, 30, &c. successively, you will have the Distance of the Eye, *viz.* nR given for projecting all the Meridians at an equal Distance upon the *Map*, and which therefore may be entitled the GLOBULAR PROJECTION, or MAP OF THE WORLD.

For Example; suppose m = 10, and p = 90, then $m : p :: 1 : 9$, therefore $n = \frac{c}{sp - R} = 1,7496$; and therefore nR = 17496, the Distance of the Eye for projecting the Meridian of 10 Degrees, and so for the rest.

If we suppose the Eye at an *infinite Distance*, then all the Meridians are projected (by the *parallel visual Rays*) into *Ellipses*; and the Distance of any Star from the *horizontal Line* will be the *Sine of its Declination*; and Radius will be to the Sine of Right Ascension, as the Cosine thereof is to the *Distance of the Star from the vertical Line* in the Projection. This Projection is of little Use in *Geography*, but of very great

great Uſe in *Aſtronomical Sciences*, and is known by the Names of ANALEMMA, and *Orthographical* PROJECTION of the SPHERE.

There are many other Projections of the Sphere, &c. to which the Rules of Perſpective may be applied, but ſuch Applications I ſhall leave to exerciſe the Ingenuity of my young Reader; it muſt ſuffice that I have here given him the Cue.

APPENDIX.

Concerning the Difference between *Optical* and *Perſpective* PLANES; and of the APPEARANCES of OBJECTS upon them.

UPON Conſideration of that vulgar Error of repreſenting a COLONADE of PILLARS in a *Front View*, all of an EQUAL SIZE, being found in all the voluminous Authors on Perſpective, both Ancient and Modern, I was willing to precaution the young Artiſt againſt it, as it is contrary to the Nature of Perſpective, and a Diſgrace to the Profeſſors of the Science; ſome of whom, to my Knowledge, have yet been ſo ignorant of its Principles, that they have not ſcrupled to defend ſo abſurd a Poſition. And indeed if you ſay, that PILLARS in any View muſt have their *Perſpectives* unequal, but thoſe of PILASTERS muſt in a front View be all *equal*, you find ſo few that can aſſign (or, indeed, ſee) the Reaſon of it, as too plainly ſhews how little the very firſt Rudiments of Perſpective are underſtood by People in general.

I mention the first Rudiments, because such Mistake is evidently owing to not attending to the Difference between the Appearance of Objects upon the *Optical, and the perspective Plane*, on which they are viewed; nor do they advert to the Difference that will arise in the perspective Appearance of Objects upon the same Plane, upon Account of their different Number of Dimensions; all which Considerations were thought *too elementary* to be allowed a Place even in the Beginning of a Treatise of Perspective, and are therefore reserved for an *Appendix* only.

This Affair may be thus represented and explained; Let O be the Center of a Circle ABC, placed in the Ground Right Line OF, *directly* before the Eye at I (see FIG. 12.) and let *(o)* be the Center of a Circle in an oblique View equal to it, at a Distance *O o* in the same Line; with the Radius IO describe an Arch OP, which will denote the Section of Part of the *optical Plane*, for such a Plane must be a *Cylinder*. Then draw any Right Line GP parallel to OF, and it will be the Section of a *perspective Plane*, which is always a *plain Surface*. Join I*o*, and at right Angles thereto, draw the Diameter DE; lastly, draw the Lines AI, CI; *a* I, *c* I; also DI crossing OF in M; and IE, cutting OF in F, when produced.

Now the Optic Angle AIC subtended by the Circle in the *direct View* is to the Angle DIE which it subtends in the *oblique View*, as I *o* to IO, or inversely as their Distances from the Eye I, as is evident from the PRINCIPLES of OPTICS; therefore the Appearance of the Diameter DE upon the *optic Plane* will be LP which is

 less

leſs than DE (= AC) in the ſame Ratio of IO to I *o*. From whence it appears, *that all Objects leſſen in Appearance upon an* Optic Plane, *in Proportion as their Diſtance from the Eye increaſes.*

But with regard to the *perſpective Plane* GH, juſt the reverſe will appear; for this Plane will cut the viſural Rays IA, IC in G and N; and the Viſuals DI, EI, in K and H. Hence GN will be the Perſpective of AB; and KH the Perſpective of the equal Line DE in the oblique View; and ſince GH is parallel to AF, we have KH : GN : : MF : AC (= *ac*) but MF is greater than *ac* or AC; *therefore* KH *the Perſpective of the Diameter of the Circle in the oblique View is greater than* GN *the Perſpective of the ſame Diameter in the direct View.*

Hence alſo it is evident that KH being the Perſpective of the Diameter of a Pillar upon the Circle in the ſide View at *(o)* and GN the Perſpective of the Diameter of an equal Pillar in the direct View at O, *the Perſpective of theſe equal* Pillars *will be unequal, and the Perſpective of the* Pillar *viewed obliquely is always greater than the perſpective of the ſame* Pillar *placed directly before the Eye.*

But Pilasters, being conſidered as Planes, have but *two Dimenſions*; and being placed in the Front Line AF, muſt have all their Baſes AC, *ac*, in the ſame Line; and the Perſpectives of theſe Lines will be GN, *kh*; and they will be equal to each other; for it is GN : *kh* : : AC : *ac*; but AC = *ac*, therefore GN = *kh*. *And therefore alſo Pilaſters or Planes ſtanding upon equal Baſes* AC, *ac*, *any where in the front Line* AF, AF, *and having their perspective Bases* GN, *kh*, *equal, will have their own Perspectives every where equal likewise.*

And from hence it appears that all Plane Surfaces whatever placed in a front Wall, or Plane, will have in their Perspectives no Change of Figure at all, a *Square*, a *Parallelogram*, a *Triangle*, a *Pentagon*, a *Circle*, an *Ellipsis*, &c. will be all the *same Figures* on the perspective Plane, and perfectly similar to the Originals; and this will hold good in every Part of such a Plane in Front as well above and below the Horizon, as on each Side the Eye.

Hence it is that the Places of the Sun, Moon, &c. and of their Images seen by Reflection, are equidistant from the Horizon of the Picture as well as from the Horizon of the Place of the Observer. Hence likewise if a BIRD were to fly perpendicularly upwards or downwards, its perspective Appearance would be always the same.

I might further observe, that as the Eye moves farther from the Plane of the Picture, the Perspectives of Objects encrease, and diminish as the Eye approaches the said Plane. Also that if ABC (FIG. 12.) be a Globe in direct View, and MDE an equal Globe in a side View, but in the same front Line AF, then the Perspective View of the former will be a *Globe* or Sphere, but that of the latter will be a *prolate Spheroid.* And many other Things will easily be accounted for by Skill in this Science, which often seem mysterious to those who understand not Perspective.

FINIS.

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